

**Department of Mathematics**  
**Howard University**  
**Math 005 & 006 College Algebra 1 Final Examination**  
**December 6, 2016**

The examination consists of 15 questions on 2 pages with a total of 200 points.  
Answer all questions. **Calculators are not allowed.**

1. [10 points] Factor completely.

(a)  $3x^3 - 7x^2 + 18x - 42$       (b)  $2x^2 + 5x - 3$

2. [10 points] Perform the indicated operation and simplify. Leave your answer in factored form.

$$\frac{x}{x^2 + 5x + 6} - \frac{2}{x^2 + 3x + 2}$$

3. [10 points] Given that  $x$  and  $y$  are positive, simplify

$$(4x^4y^{-2})(-3x^5y^{-4})$$

Write your answer so that each variable appears only once, and use positive exponents only.

4. [10 points] A boat travels 50 miles downstream in the same time that it takes to go 30 miles upstream. The speed of the stream is 3 miles per hour. Use algebra to find the speed of the boat in still water.
5. [30 points] Find all real solutions, if any, of each equation.

(a)  $(x - 1)(x + 1) = 5(x - 1)$       (b)  $\frac{14}{x+2} - \frac{1}{x-4} = 1$

(c)  $x - 5 = \sqrt{x + 7}$       (d)  $5^{1-x} = \frac{1}{25}$

(e)  $\log x + \log(x + 1) = \log(x + 3)$

6. [20 points] Solve each inequality, and give the solution in interval notation. Also, graph the solution set.

(a)  $16 < 7 - 3x \leq 31$       (b)  $|7x - 5| \leq 1$

7.[10 points] Write each complex number in standard form  $a + bi$ .

(a)  $(5 + 2i)(4 - 3i)$                       (b)  $\frac{-5+10i}{3+4i}$

8.[10 points] Find the standard form of the equation of the circle that has a diameter with endpoints  $(-4, 3)$  and  $(6, 3)$ .

9.[15 points] A line has equation  $5x + 3y = 15$ .

(a) Find the slope and  $y$ -intercept of the line.

(b) Find an equation for a perpendicular line that passes through the point  $(3, -4)$

(c) Find an equation for a parallel line that passes through the point  $(3, -4)$ .

10.[10 points] A circle has equation  $x^2 + y^2 - 4x - 6y + 9 = 0$ .

(a) Find the center and radius of the circle.

(b) Graph the circle.

11.[15 points] Consider the function  $f(x) = \begin{cases} x+1 & \text{if } -1 \leq x < 0 \\ -x+1 & \text{if } 0 \leq x \leq 1 \end{cases}$ .

(a) Find  $f(-1)$ ,  $f(0)$ , and  $f(1)$ .

(b) Graph the function.

(c) State the domain and range of the function in interval notation.

12.[20 points] Consider the quadratic function  $f(x) = -x^2 - 2x + 3$ .

(a) Find the vertex of the graph of the function.

(b) Find the  $x$  and  $y$  intercepts.

(c) Graph the function and show the vertex and intercepts.

(d) State the range of the function in interval notation.

13.[10 points] Let  $f(x) = \sqrt{x+1}$  and  $g(x) = 3x$ .

(a) Find  $\left(\frac{f}{g}\right)(x)$  and give the domain in interval notation.

(b) Find the composite function  $(g \circ f)(x)$  and give its domain in interval notation.

14.[10 points] Find the inverse  $f^{-1}(x)$  of the function  $f(x) = \sqrt{x-3}$  and state the domain and range of  $f^{-1}(x)$  in interval notation.

15.[10 points] Write  $3 \ln x + \ln y - \frac{1}{3} \ln z - 2 \ln w$  as a single logarithm. Assume that all variables are positive.