

MATH 006 COLLEGE ALGEBRA I FINAL EXAM (SPRING 2006)

EXAM CONSISTS OF TWO PAGES TOTTALLING 200 POINTS. SHOW YOUR WORK.

1) (18 pts) Simplify each expression; assuming $x \geq 0$ and $y > 0$:

(a) $4\sqrt{12} + 5\sqrt{27}$ (b) $\frac{\sqrt{2x}\sqrt{5xy^2}}{\sqrt{10y}}$ (c) $\frac{\frac{z+4}{3}}{\frac{1}{4} + \frac{2}{z}}$

2) (56 pts) Find all real solutions, if any, of each equation or inequality:

(a) $\frac{4-2x}{3} + \frac{1}{6} = 2x$ (b) $x(1+x) = 6$
 (c) $\sqrt{2x-1} = x-2$ (d) $3x^3 + 5x^2 - 3x - 5 = 0$
 (e) $|1-2x| + 1 = 4$ (f) $|3x+1| > 10$
 (g) $-3 \leq \frac{5-3x}{2} \leq 6$ (h) $\frac{x+2}{x(x-5)} \leq 0$

3) (10 pts) Two pumps of different sizes can empty a fuel tank in 5 hours. The larger pump can empty this tank in 4 hours less than the smaller. If the larger pump is out of order, how long will it take the smaller pump to the job alone?

4) (20 pts) Consider the pair of points $P = (-2, 2)$ and $Q = (1, 4)$.

- (a) Find the distance between P and Q .
- (b) Find the midpoint M of the line segment connecting P and Q .
- (c) Find an equation of the line L_1 containing P and Q .
- (d) Find an equation of the line L_2 containing the midpoint M found in (b), which is perpendicular to line L_1 found in (c).
- (e) Find an equation of the line L_3 through the origin $(0, 0)$, which is parallel to the line L_1 found in part (c).

5) (14 pts) Consider the circle described by the equation

$$x^2 + y^2 + 4x - 6y - 3 = 0$$

- (a) Find its center and radius. (b) Sketch its graph.

6) (10 pts) The resistance (in ohms) of a circular conductor varies directly with the length of the conductor and inversely with the radius of the conductor. If 50 feet of conducting wire with a radius of 6×10^{-3} inch has a resistance of 10 ohms, what would be the resistance of 100 feet of the same wire if the radius is increased to 7×10^{-3} inch?

(over)

7) (12 pts) Find the following for the function f given by

$$f(x) = \frac{x^2}{x+1}$$

- (a) $f(2)$ (b) $f(-2)$ (c) $f(-x)$
(d) $-f(x)$ (e) $f(x-2)$ (f) $f(2x)$

8) (10 pts) Sketch the graph of the piecewise defined function given by

$$g(x) = \begin{cases} x - 1 & -3 < x < 0 \\ -2x - 1 & x \geq 0 \end{cases}$$

9) (10 pts) A farmer with 2000 meters of fencing wants to use all of it to enclose a rectangular plot that borders a straight highway. If the farmer does not fence the side along the highway, what is the largest area that can be enclosed?

10) (10 pts) Find the domain of the rational function f , and the equations of its horizontal, vertical, or oblique asymptotes, if any; given

$$f(x) = \frac{x+2}{x^2-9}$$

11) (30 pts) Using the graph of the function g given below:

- (a) Find the domain and range of g .
(b) Find $g(-1)$.
(c) List the intercepts.
(d) For what value(s) of x does $g(x) = -3$?
(e) Solve: $g(x) > 0$.
(f) Determine the interval(s) on which g is increasing; on which g is decreasing; on which g is constant.
(g) Sketch the graph of $y = g(x-2)$.
(h) Sketch the graph of $y = g(x) + 2$.
(i) Sketch the graph of $y = 2g(x)$.

