Precalculus MATH 007
Fall 2012 Final Exam
Howard University
Department of Mathematics
4 December 2012

Instructions: This exam consists of 14 questions. Show all your work. No Work, No Credit. Number each problem solution on a separate page in the exam booklet. Use front and back. No more than two problems per page.

10 Points (1) Use the Laws of Logarithms to combine the expression $\log x + 2\log y - \log z$.

15 Points (2) Solve the logarithmic equation for the variable x.

$$\log_3(x-4) + \log_3(x+4) = 2.$$

20 Points (3) Solve the exponential equation for the variable x.

$$e^{2x} - 3e^x - 10 = 0.$$

15 Points (4) Sketch a triangle that has an acute angle θ, and find the other five trigonometric ratios of θ, given $\tan\theta = \frac{1}{2}$.

NOTE: You may do either problem 5A or problem 5B.

20 Points (5A) Identify the domain, x and y intercepts, **horizontal, vertical, and slant asymptotes** (if any);

$$\text{a) } r(x) = \frac{x^3+3x^2}{x^2-4}, \qquad \text{b) } s(x) = \frac{2x-4}{x^2+x-2}.$$

20 Points (5B) Maximize the function $F(x,y) = 3x + 5y$ subject to the following constrictions:

$$x \geq 0, \qquad y \geq 0, \qquad x+y \geq 2, \qquad 2x+3y \leq 12, \qquad 2x+3y \leq 12$$

20 Points (6) Without the use of a calculator, find the exact value of the following;
(a) $\cos(75°)$, (b) $\tan^{-1}\left(\tan(\frac{\pi}{4})\right)$, (c) $\sec\left(\frac{11\pi}{4}\right)$, (*d*) $\tan\left(\frac{5\pi}{12}\right)$

15 Points (7) Given the trigonometric function below find; (a) the amplitude, period and phase shift. (b) Graph the function, showing one complete period.

$$y = 2\sin\left(\frac{2}{3}x - \frac{\pi}{6}\right)$$

In problems 8 and 9, A, B, C denote the angles of a triangle and a, b, c denote the sides opposite these angles, respectively.

10 Points (8) Give an example of a triangle, where one angle is given, that cannot be solved using only the Law of Sines. Use the Law of Sines to solve the triangle $a = 12, b = 20$, and $A = 30°$. You may use a calculator for this problem, but you must show all of your work.

15 Points (9) Solve the triangle with sides $a = 8$, $b = 10$, and $c = 3$. You may use a calculator for this problem, but you must show all of your work.

10 Points (10) Show that:

$$\sin\theta \tan\theta \sec\theta + 1 = \sec^2\theta$$

10 Points (11) Show that:

$$(\cos\alpha\cos\beta - \sin\alpha\sin\beta)^2 + (\sin\alpha\cos\beta + \cos\alpha\sin\beta)^2 = 1.$$

20 Points (12) A 600 ft. guy wire is attached to the top of a communications tower. If the wire makes an angle of 65° with the ground, how tall is the communications tower?

10 Points (13) The carrier wave for an FM radio signal is modeled by the function $y = a\sin(2\pi(9.15 \times 10^7)t)$, where t is measured in seconds. Find the period and frequency of the carrier wave.

20 Points (14) Write the augmented matrix of this system. Then solve by using the **Row-Echelon form**.

$$\begin{cases} 5x - 2y - 4z = 3 \\ 3x + 3y + 2z = -3 \\ -2x + 5y + 3z = 3 \end{cases}$$