

HOWARD UNIVERSITY

Final Examination

College Algebra II (Math-010) Fall 2009

Total points: 200

SHOW ALL YOUR WORK

1. [10 pts] The function $f(x) = \frac{2x-3}{x+4}$ is a one-to-one.
 - a) Find the domain of the function f .
 - b) Find its inverse.
 - c) Use the inverse found in b) to determine the range of the function f .

2. [15 pts] Solve each equation for x .
 - a) $\log_3(x+1) + \log_3(x+4) = 2$
 - b) $9^x - 3^{x+1} + 1 = 0$

3. [10 pts] How long would it take for an investment to triple its value if it earns 7% interest compounded continuously?

4. [10 pts] John will be buying a used car for \$20,000 in 4 years. How much should he ask his parents for now so that, if he invests it at 6% compounded quarterly, he will have enough money to buy the car?

5. [10 pts] The population of a Midwestern city follows the exponential law. If the population decreased from 1,600,000 to 1,500,000 from 2006 to 2008, what will be the population in 2011?

6. [15 pts]
 - a) Find the vertex, focus, directrix, latus rectum, and axis of the parabola with equation $y^2 - 2y = 8x - 1$.
 - b) Graph the equation.

7. [15 pts]

- a) Find the center, major axis, foci, vertices, the intersection points with the minor axis of the ellipse whose equation is given below.

$$4x^2 + 3y^2 + 8x - 6y = 5$$

- b) Graph the equation.

8. [10 pts] Solve the system of nonlinear equations below:

$$\begin{cases} y^2 - x^2 + 4 = 0 \\ 2x^2 + 3y^2 = 6 \end{cases}$$

9. [10 pts] Use Cramer's rules to solve the system

$$\begin{cases} 2x + y - z = 3 \\ -x + 2y + 4z = -3 \\ x - 2y - 3z = 4 \end{cases}$$

10. [10 pts]

- a) The matrix $\begin{bmatrix} -4 & 1 \\ 6 & -2 \end{bmatrix}$ is nonsingular. Find its inverse.

- b) Use the inverse found in question 10) a) to solve the system of linear equations

$$\begin{cases} -4y + y = 5 \\ 6x - 2y = -9 \end{cases}$$

11. [10 pts] If $A = \begin{bmatrix} -2 & 3 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & 3 \\ -3 & 0 & 4 \end{bmatrix}$ determine each of the following matrices whenever possible.

- a) AB

- b) BA

12. [10 pts] Maximize $z = 2x + 4y$ subject to $x \geq 0, y \geq 0, 2x + y \geq 4, x + y \leq 9$.

13. [10 pts]

a) Find a formula for the n^{th} term of the arithmetic sequence whose 4th term is 3 and 20th term is 35.

b) Give a recursive formula for the sequence.

14. [10 pts] Find the sum of all odd numbers from 7 to 493 inclusive.

15. [15 pts] The sequence given below is geometric.

$$3, \frac{6}{5}, \frac{12}{25}, \frac{24}{125}, \dots$$

a) Determine its common ratio.

b) Determine its n^{th} term.

c) Use the answer in b) to determine its fifty third term.

16. [10 pts] What is the number of subsets of the set {a, b, c, d, e, f}?

17. [10 pts] Find the coefficient of x^4 in the expression of $(2x - 3)^6$?

18. [10 pts] A manufacturer of mainframe computers is trying to win a large contract with a client. The manufacturer is going to send to the client's facilities 5 salespersons, 7 technicians, and 3 software engineers. In how many ways can these 15 people be selected if the manufacturer has available 10 salespersons, 9 technicians, and 7 software engineers?

19. [BONUS 5 pts]

A radioactive substance is decaying exponentially. The amount of substance is reduced from 800 grams to 400 grams after 4 days. How much of the substance would remain after 10 day? (Hint: use the formula $A(t) = A_0 e^{kt}$, $k < 0$)

20. [BONUS 5 pts]

How many different 11-letters words (real or imaginary) can be formed from the letters in the word MATHEMATICS?