

The Final Examination For Math 026 Applied Calculus Fall 2009

No calculator for this examination. There are 16 problems. Problems 1 to 8 each one has 13 points and problems 9 to 16 each one has 12 points. The total of 200 points. Show all your works.

1. Find the following limits:

(a)  $\lim_{x \rightarrow 2} \frac{(x^2-4)}{(x^2+x-6)}$  (b)  $\lim_{x \rightarrow \infty} \frac{2x^2}{10+100x+1000x^2}$

2. Let  $f(x) = x + \frac{1}{x}$ . Find  $\frac{f(x)-f(1)}{x-1}$  and simplify. (b) By using a definition for the derivative find  $f'(1)$ .

3. Find an equation of the tangent line to the graph of function  $f(x) = \ln(2x + 9)$  at  $x = -4$ .

4. Use the techniques of differentiations to find the derivative of the following functions and simplify the result:

(a)  $f(x) = \frac{x^2+1}{1-x^2}$ , (b)  $g(x) = \frac{e^x+e^{-x}}{e^x-e^{-x}}$ , (c)  $h(x) = \ln((x^2 + 1)\sqrt{x^2 + 1})$

5. Let a function  $f$  be given by  $f(x) = 3x^4 - 8x^3 - 18x^2$ . (a) Find the intervals on which the function is increasing or decreasing. (b) Find the critical numbers and the critical points and classify each one as a relative maximum, relative minimum or neither.

6. Determine where the function  $f(x) = x^4 - 4x^3 + 10$  is concave up or concave down and find the inflection points.

7. A manufacturer estimates that when  $x$  units of a particular commodity are produced, the total cost will be :

$C(x) = \frac{1}{8}x^2 + 3x + 98$  dollars and that  $P(x) = \frac{1}{3}(75 - x)$  dollars per unit is the price at which all  $x$  units will be sold.

(a) Find the marginal cost. (b) Use marginal cost to estimate the cost of producing the 9th unit. (c) what is the actual cost of producing the 9th unit ?. (d) Find the revenue function in terms of  $x$ .

8. A city recreation department plans to build a rectangular playground having an area of 6581 square meters and surrounded by a fence. How can this be done using the least amount of fencing?. Justify your answer.

9. Use implicit differentiation to find  $\frac{dy}{dx}$  if  $x^2y^4 + x^3y - y^3 + x - 2 = 0$ .

10. Find the absolute maximum and the absolute minimum of the function  $f(x) = 4x + \frac{1}{x}$  over the interval  $[\frac{1}{8}, 1]$ .

11. Evaluate the integral  $\int 5xe^{-x^2} dx$ .

12. Graph the functions  $f(x) = -x^2 + 2$  and  $g(x) = x^2$  and find the area of the region between them.

13. Find the first partial derivatives and the second partial derivatives of the function:

$$f(x, y) = \frac{x-y}{x+y}, \text{ and simplify the results.}$$

14. Find the integral  $\int_1^2 (\frac{1}{x} + \frac{2}{x^2} + 3) dx$ .

15. Find the critical points of the function with two variables

$$f(x, y) = x^2y - \frac{1}{2}y^2 - 4x^2 + 10$$

and classify each one as a relative maximum, a relative minimum or a saddle point.

16. Evaluate the double integral  $\int_0^3 \int_1^2 (2xy + 2y + 1) dx dy$ .