FINAL EXAM MATH 026 APPLIED CALCULUS FALL 2010

Instructions. Answer all of the questions #1 - 10 which are worth 14 points each. Choose 3 of the questions #11 - 16 which are worth 20 points each.

The total exam is worth 200 points. Write and number each problem on a **separate page in the exam booklet. Show your work.** No work/No credit !!

1. (a) Determine the following limit:

   $\lim_{x\to 1} \frac{x^2-1}{x^2+3x-4}$

   (b) Determine the following limit:

   $\lim_{x\to\infty} \frac{2x^5+3x^2-5}{7x^5-3x^3-5x}$

   (c) Determine if the following function is continuous at x=5.

   $$f(x) = \begin{Bmatrix} x^2-15 & x < 5 \\ \frac{x^2-25}{x-5} & x > 5 \end{Bmatrix} \text{ and } f(5) = 0. \qquad \text{Explain your answer.}$$

2. Use the definition of the derivative to find $f'(2)$ where $f(x) = 2x^2 + 10$

3. Find the derivative of the following functions and simplify the results:

   (a) $f(x) = \ln\left(2x^2 + 5x\right)$

   (b) $g(x) = \frac{e^x+e^{-x}}{2x^2}$

   (c) $h(x) = \ln\left(e^x\sqrt{(5x^2+2)}\right)$

4. Use implicit differentiation to find $\frac{dy}{dx}$ if $x^2y - y^2x + 4y + 2x = 10$

5. Use logarithmic differentiation to find the derivative of the following function.

   $f(x) = \frac{\sqrt{(x^2+3)(e^x)}}{5x^3}$

6. Find the equation of the tangent line at $x = 0$ to the function
$f(x) = \ln\left(x^2 + 5x + 1\right)$

7. The price $S(x)$ at which x units of a particular commodity can be sold is given by $S(x) = 300 - 2x$, and the total cost $C(x)$, of producing the x units is given by $C(x) = 3x^2 + 10x + 75$

(a) Find the revenue function $R(x)$

(b) Find the profit function $P(x)$.

(c) Determine the level of production x where $P(x)$ is maximized.

8. A company's annual profit after t years is $P(t) = t^3 - 9t^2 - 48t$ million dollars, for $t \geq 0$.

(a) Determine where the function is increasing and where it is decreasing.

(b) Determine all relative maximum and minimum points.

(c) Determine where the function is concave up and where it is concave down and find all points of inflection.

(d) Sketch the graph of the function.

9. Evaluate the following integrals:

(a) $\int \frac{x^2+2x-1}{x^2} dx$

(b) $\int_1^5 x + \frac{1}{x}\, dx$

10. Evaluate the following integrals:

(a) Find : $\displaystyle\int_0^3 \frac{2x}{(x^2+16)^{\frac{3}{2}}} dx$

(b) Use integration by parts to find : $\int xe^{2x} dx$

DO ANY 3 OF THE FOLLOWING PROBLEMS:

11. Find the following partial derivatives
f_x, f_y, f_{xx}, f_{yy}, f_{xy}, f_{yx} for the function $f(x,y) = 2xy^2 - 3x^2y + x^2$

12. Find the particular solution of the differential equation $\frac{dy}{dx} = 4x + 1$
for $y = 4$ when $x = 1$

13. The Demand function for a given commodity is $D(p) = 81 - p^2$,
where is the price in dollars $(0 \leq p \leq 9)$.
Compute the elasticity of demand and determine whether the demand is elastic, inelastic, or of unit elasticity at the indicated price.
a). $p = 3$

b). $p = 6$.

14. Find all critical points of the function $f(x,y) = xy^2 - 6x^2 - 3y^2$ and classify each as a relative minimum, relative maximum or a saddle point.

15. For several weeks, the highway department has been recording the speed of freeway trafic flowing past a certain down town exit. The data suggests that between 1:00 and 6:00 on a normal weekday, the speed of traffic at the exit is approximately $S(t) = t^3 - \frac{21}{2}t^2 + 30t + 18$ miles per hour, where t is the number of hours past noon.

a) At what time between 1:00pm and 6:00pm is the traffic moving the fastest?

b). At what time between 1:00pm and 6:00 pm is the traffic moving the slowest?

16. Find the area between the curves $f(x) = x^2 + 5$ and $g(x) = 3x + 15$