# Calculus I - Final Examination - Fall 2006

No calculators. Please explain your reasoning.

Tuesday December 12, 4:00 - 6:00 PM

Do any 10 questions. Each question is worth 20 points.

**1.** A spherical balloon is being blown up at a the rate of 0.03 $m^3$/sec. How fast is the radius increasing when the volume is $0.35m^3$? How fast is the surface area increasing when the volume is 0.35 $m^3$?

**2.** Differentiate each of the following

- $f(x) = x^5 + 4^x + \sin 3x + \tan 2x + e^x + \ln x$
- $y = \frac{1}{3x^3} + \frac{1}{x\sqrt{x}} - \ln x^2$
- $y = \sin^2(\ln x)$
- $y = x^3 e^{-2x}$

**3.** Use only the definition of the derivative to find $g'(3)$ where $g(x) = 3x^2 + 2x + 1$

**4.** Sketch a possible graph for a function, $h = h(x)$, with all the following characteristics:

- $h$ is defined for all real numbers
- $h' > 0$ for all $x$
- $h'' < 0$ for all $x$
- $h(3) = 2$
- $h'(3) = 1$

The graph of $h$ has one $x$-intercept. In what interval must it be located?

**5.** Find and classify the relative extrema of the function

$$f(x) = e^{\sin x}.$$

**6.** Evaluate the integrals

**a.** $\int x^3 + x^{-3} + \cos x + e^{2x} + \frac{3}{1+x^2}\, dx$

**b.** $\int \frac{\sin(\frac{5}{x})}{x^2}\, dx$

**c.** $\int_{-\ln 3}^{\ln 3} \frac{e^x}{e^x+4}\, dx$

**d.** $\int_{\pi/12}^{\pi/9} \sec^2 3\theta\, d\theta$

**7.** Find the point on the curve $y = \sqrt{x}$ closest to the point $(2, 0)$?

**8.** Find the relative and absolute maxima and minima points of the curve $f(x) = 2x^3 - 3x^2 - 12x$ on the closed interval $[-2, 3]$. Are there any points of inflection in $[-2, 3]$?

**9.** Find $A$ so that

$$f(x) = \begin{cases} Ax^2, & x < 2 \\ x^3, & x \geq 2 \end{cases}$$

is continuous for all real numbers $x$. Will this $f(x)$ be differentiable for all real numbers? Why?

**10.** Find, with explanations, the following limits

- $\lim_{x \to 0} \frac{\sin 3x}{\sin 5x}$
- $\lim_{x \to 4} \frac{\sqrt{x}-2}{x-4}$
- $\lim_{x \to 0} \frac{\sin x}{x+x\cos x}$

**11.** Find, with your reasoning,

$$\frac{d^{100}}{dx^{100}}(xe^x).$$

**12.** State the following theorems and explain with one sentence each why they are important.

- The mean value theorem
- the fundamental theorem of calculus