Calculus I

Math 156 **Fall 2016** **Final Exam A**

Show All Work. No work = no credit. Show respect for your professor by respecting the Howard University Academic Integrity code. No phones, laptops, smart watches.

1. (10 pts) Find the following limits.

   (a) $\lim_{x\to 0}\left(\frac{1}{e^x-1}-\frac{1}{x}\right)$

   (b) Evaluate without using L'Hôpital's Rule. $\lim_{x\to 0}\frac{1-\cos 2x}{x^2}$

2. (20 pts) (a) Use the Squeezing Theorem to compute $\lim_{x\to 0} 3x^4\cos\left(\frac{2}{x}\right)$.

   (b) Find the total area between the curve $y = x^2 - x$ and the $x-$axis in the interval [0,2].

3. (10 pts) Explain why the function below is discontinuous at $x = -5$, then determine a value of $f(-5)$ that would make f continuous at $x = -5$.

$$f(x) = \begin{cases} x^2+12x+35 & \text{if } x < -5 \\ 3 & \text{if } x = -5 \\ -x^2-12x-35 & \text{if } x > -5 \end{cases}$$

4. (10 pts) Sketch the graph of a function that satisfies all of the following conditions:

$$\lim_{x\to 3} f(x) = -\infty \quad \lim_{x\to\infty} f(x) = \infty \quad \lim_{x\to-\infty} f(x) = 5 \quad \lim_{x\to 0^+} f(x) = -\infty \quad \lim_{x\to 0^-} f(x) = \infty$$

5. (10 pts) Compute the first and second derivatives of $k(x) = x^{5/2} - 3\sqrt[4]{x} - \frac{2}{x^5} + e^{3x}$.

6. (10 pts) Find two positive integers such that the sum of the first number and four times the second number is 1000 and the product of the numbers is as large as possible.

7. (40 pts) Compute the derivative of the following functions:

   (a) $k(x) = \frac{(x^4-5)(e^x-7x)}{x^2+3}$ (b) $m(x) = \cos(\sin(x))$

   (c) $h(x) = \tan^5(x) - 3\tan(x^2) + 5e^{(-9x^2)}$ (d) $g(x) = 2x\ln(x) - x + 8^x$

8. (10 pts) A pebble is dropped into a pond, and the resulting ripple travels 3 ft/sec. How fast is the area inside the ripple increasing when the ripple is 8 feet in diameter?

9. (15 pts) Find the following indefinite integrals.

   (a) $\int \sec^2 y\, dy$ (b) $\int \frac{1+y^2}{\sqrt{y}}\, dy$ (c) $\int y\sqrt{y^2+1}\, dy$

10. (10 pts) Consider the curve defined by $e^{xy} - x^3y^2 = x + \sin(y)$.

    (a) Find $\frac{dy}{dx}$.

    (b) Which of the points $(0,1)$ or $(1,0)$ lies on this curve?

    (c) Find an equation of the tangent line to the curve at the point you found in 10b.

11. (10 pts) (a) Find the derivative of the function $F(x) = \int_x^{x^2} e^{6t^2}\,dt$.

    (b) Write as a single integral in the form $\int_a^b f(x)\,dx$:

    $\int_{-2}^{2} f(x)\,dx - \int_{5}^{2} f(x)\,dx - \int_{-2}^{-1} f(x)\,dx.$

12. (15 pts) Evaluate the following definite integrals.

    (a) $\int_0^{\pi/3} (1 + \sin^2 x)\cos x\,dx.$ (b) $\int_0^1 (u+2)(u-5)\,du$ (c) $\int_2^2 \sqrt{3+x^4}\,dx.$

13. (10 pts) A person leans out of a high window and throws a ball straight up so that t seconds later its height above the sidewalk is $192 + 64t - 16t^2$ feet.

    (a) When does the ball reach its maximum height above the sidewalk, and what is that height?

    (b) When does the ball hit the sidewalk?

14. (10 pts) Find two positive integers such that the sum of the first number and four times the second number is 1000 and the product of the numbers is as large as possible.

15. (10 pts) Consider the family of polynomials given by the equation $f(x) = x^4 - 4x^2 + 4$.

    (a) Where is $f(x)$ the curve increasing on $(-5, 5)$?

    (b) For what values of x does the curve have minimum points?

    (c) For what values of x does the curve have an inflection point?

16. (10 pts) (a) Use a local linear approximation to estimate $\sqrt[3]{63}$.
    (b) Consider the graph of $G'(x)$ is shown below. Sketch a possible graph of $G(x)$.