

Name: _____

Show all work and justify your answers (No credit without work shown). Respect the Howard University Academic Integrity code.

Solve *all* of the questions 1 through 5.

1. (20 points) (a) Find $\frac{d}{dx} \int_1^x \frac{t-1}{t+1} dt$.

(b) Use logarithmic differentiation to find the derivative of $f(x) = x^{\cos x}$

(c) Find the derivative of:

(i) $\ln(x - \sqrt{x^2 - 1})$. Make sure your answer is simplified as much as possible.

(ii) $y = x^3 e^{\cos(5x)}$

2. (20 points) (a) Let $f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0. \end{cases}$

(i) Find $f'(x)$ when $x \neq 0$ using differentiation techniques.

(ii) Find $f'(0)$ using the definition of the derivative.

(iii) Is $f'(x)$ continuous at $x = 0$? Justify your answer.

(b) Find values of the constants a and b , if possible, that will make the function f continuous everywhere.

$$f(x) = \begin{cases} x^2 + 5, & \text{if } x > 2, \\ b(x+1) + a, & \text{if } -1 < x \leq 2 \\ 2x^3 + x + 7, & \text{if } x \leq -1 \end{cases}$$

3. (20 points) Evaluate the following limits:

(a) $\lim_{y \rightarrow \infty} \frac{4+y}{9-y^2}$

(b) $\lim_{t \rightarrow 3^+} \frac{t^2 - 9}{\sqrt{t} - 3}$

(c) $\lim_{x \rightarrow 0} \frac{e^x - x - 1}{\cos x - 1}$

(d) $\lim_{x \rightarrow \infty} x^2 e^{-x}$.

4. (20 points) Evaluate each integral.

(a) $\int \left(\frac{1}{4x} + 2e^{-2x} - \frac{5}{x\sqrt{x}} \right) dx$

(b) $\int \frac{4x + 8}{(x^2 + 4x + 5)^2} dx$

(c) $\int \frac{e^{3/x}}{x^2} dx$

(d) $\int_{\pi/4}^{\pi/2} \frac{\cos x}{\sin^2 x} dx.$

5. (20 points) A sign chart is provided for the first and second derivative of a function f . Assuming that f is *continuous* everywhere, find:

Table 1: Sign Chart

Interval	Sign of $f'(x)$	Sign of $f''(x)$
$x < 0$	-	-
$0 < x < 3$	+	-
$3 < x < 5$	-	-
$5 < x < 8$	-	+
$8 < x$	+	+

- (a) the largest intervals on which f is increasing;
- (b) the largest intervals on which f is decreasing;
- (c) the open intervals on which f is concave up;
- (d) the open intervals on which f is concave down;
- (e) the x -coordinates of all inflection points, if any;
- (f) the x -coordinates of all relative maxima, if any;
- (g) the x -coordinates of all relative minima, if any.

Solve *five* of the questions 6 through 11.

- 6. (20 points) Use implicit differentiation to find an equation of the tangent line to the graph of $ye^x + xe^y = 4$ at $(0, 4)$.
- 7. (20 points) The radius of a circular oil slick expands at a rate of 2 m/min. How fast is the area of the slick increasing when the radius is 25 m?

8. (20 points) (a) (i) Find the equation of the tangent line to the graph of $y = \frac{1}{x^2+1}$ at $x = 1$.
(ii) Where on the curve of y does the tangent line have the greatest slope?
(b) Find the linear approximation of $f(x) = \ln x$ at $a = 1$. Use the linear approximation to approximate $\ln 1.01$.
9. (20 points) (a) State Rolle's theorem.
(b) Let $f(x) = x^3 - 3x^2 + 2x$. Verify that the hypotheses of Rolle's theorem are satisfied by $f(x)$ in $[0, 2]$. Find the point in $[0, 2]$ where the conclusion of Rolle's theorem is satisfied.
10. (20 points) A container with square base, vertical sides, and open top is to be made from 1000 ft^2 of material. Find the dimensions of the container with the greatest volume.
11. (20 points) The function $s(t) = te^{1-t}$, $t \geq 0$, describes the position of a particle moving on a coordinate line, where s is in meters and t is in seconds.
- (a) Find the velocity and acceleration functions.
(b) Find the position, velocity, and acceleration at time $t = 1$.
(c) At what time(s) is the particle stopped?
(d) When is the particle speeding up? Slowing down?
(e) Find the total distance traveled by the particle from time $t = 0$ to $t = 3$.