HOWARD UNIVERSITY
Department of Mathematics
Final Examination
MATH 157 — Calculus II
Fall 2014

Answer any ten (10) problems. Show all work.

1. (20 pts) Determine if the following integral converges or diverges. If the integral converges determine its value.

$$\int_{-2}^{1} \frac{e^{\frac{1}{x}}}{x^2} dx.$$

2. (20 pts) Use a trigonometric substitution to evaluate the following integral

$$\int \frac{1}{x^2\sqrt{x^2-9}} dx.$$

3. (20 pts) Express the integrand as a sum of partial fractions and then evaluate the integral

$$\int \frac{y^2+2y+1}{(y^2+1)^2} dy.$$

4. (20 pts) Determine if the series $\sum_{n=0}^{\infty} \frac{3(-1)^n}{4^n} = 3 - \frac{3}{4} + \frac{3}{16} - \frac{3}{64} + \ldots,$ diverges or converges. If it converges, compute it.

5. (20 pts) Use a suitable u-substitution to evaluate the integral:

$$\int \frac{e^{4x}+2e^{2x}-e^x}{e^{2x}+1} dx.$$

6. (20 pts) Find a power series representation for the following function and determine its interval of convergence: $f(x) = \ln(x+2)$.

7. (20 pts) Find the area of the region enclosed by the curves $y = x^3$ and $y = 3x^2 - 4$.

8. (20 pts)
   (a) Evaluate the integral

$$\int \sinh(2x)\sinh(5x) dx.$$

(b) Evaluate the integral

$$\int \sin^2 x \cos^4 x dx.$$

9. (20 pts) Find the first four nonzero terms for the Taylor polynomial of $\sqrt{x}$ about $a = 1$.

10. (20 pts)
    (a) Use the Integral Test to determine if the infinite series

$$\sum_{k=1}^{\infty} \frac{k^2}{e^{k/3}},$$

converges or diverges.

(b) Determine the convergence or divergence of the following series

$$\sum_{k=1}^{\infty} \frac{8 \tan^{-1} k}{1+k^2}.$$

11. (20 pts) Use a suitable trigonometric substitution to evaluate the following definite integral:

$$\int_0^1 \frac{1}{(4-x^2)^{3/2}} dx.$$

12. (20 pts) Find the polar equation for the circle $x^2 + (y+4)^2 = 16$.

13. (20 pts) Find the area bounded by the curve $r = 5(1 + sin\theta)$.

14. (20 pts) Determine the interval and radius of convergence of the following series

$$\sum_{n=1}^{\infty} \frac{3^n}{n}(2x+3)^n.$$

15. (20 pts) Evaluate each of the following integrals

    (a) $\int \cos(3x) \sin(7x) dx.$

    (b) $\int \sin^4 x \cos^3 x dx.$

16. (20 pts) Determine whether the following series converge absolutely, converge conditionally, or diverge, giving reasons for your answers.

    a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+2}$

    b) $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{(n+1)(n+2)}$