

SHOW ALL WORK. Justify your answers!

Simplify your answers. Give exact answers whenever possible.

Each problem = 20 pts. Solve any 10 problems below. Exam total = 200 pts.

1. Find the area bounded by the curves $y = x$, $y = \tan x$, and $x = \pi/4$.
2. Find the coordinates of the centroid of the region bounded by $y = \cos x$, $y = \sin x$, $x = 0$, $x = \pi/4$.
3. Express the integrand as a sum of partial fractions and then evaluate the integral

$$\int \frac{x + 4}{x^3 + 4x} dx$$

4. Evaluate the following integrals:

(a) $\int_0^{\pi/2} \cos^3 x \sin^2 x dx$

(b) $\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

5. Evaluate the following integrals:

(a) $\int \frac{dx}{\sqrt{2x - x^2}}$

(b) $\int \cos(\ln x) dx$ *Hint:* First make a substitution, then use integration by parts.

6. Find the general term a_n of the series

$$1 + \frac{1 \cdot 2}{1 \cdot 3} + \frac{1 \cdot 2 \cdot 3}{1 \cdot 3 \cdot 5} + \frac{1 \cdot 2 \cdot 3 \cdot 4}{1 \cdot 3 \cdot 5 \cdot 7} + \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9} + \cdots$$

and determine whether the series converges.

7. Determine whether the series are absolutely convergent, conditionally convergent, or divergent (and give reasons for your answers):

(a) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

(b) $\sum_{n=1}^{\infty} \frac{(-2)^n n!}{n^n}$

8. Find the Taylor series for the function $f(x) = x^3 - 3x$ about $x = 1$.

9. Find the Maclaurin series representation of $f(x) = \tan^{-1} x$.
10. Find the radius of convergence and the interval of convergence (including endpoints where needed) of the series

$$\sum_{n=1}^{\infty} \frac{(2x-1)^n}{n3^n}$$

11. (a) Find the length of curve $y = \cosh x$ for $-1 \leq x \leq 1$.
(b) Find the area of the surface obtained when the infinite curve $y = e^{-x}$, $x \geq 0$, is rotated about the x -axis.
12. Find the resulting volume when the region between $y = e^{-x^2}$ and $x = 0$ and to the right of $y = 0$ is rotated about the y -axis.
13. Find the foci and eccentricity of the ellipse given by $\frac{(x+2)^2}{225} + \frac{(y-1)^2}{81} = 1$.
14. Determine if the sequence (a_n) , where $a_n = \frac{(\ln n)^2}{n}$, converges or diverges. If it converges, compute the limit.
15. Find the area bounded by the curve $r = 4(1 + \sin \theta)$ in polar coordinates.
16. Find the slope of the tangent line to $r = 4(1 + \sin \theta)$ at $\theta = \pi/4$.