DEPARTMENT OF MATHEMATICS

MATH 157: CALCULUS II: FINAL EXAMINATIONS.

SPRING SEMESTER: WEDNESDAY, MAY 3$^{\text{rd.}}$, 2006.

ANSWER ANY TEN [[10]] QUESTIONS

TIME: 4:00PM-6:00 PM.

1.[20pt.]

(*a*) Find the radius of convergence and the interval of convergence of the series:

$$\sum_{k=0}^{\infty} \frac{(-3)^k x^k}{\sqrt{k+1}}.$$

(*b*) Use any tests to determine whether the following series converge or diverge:

$$(i) \quad \sum_{k=1}^{\infty} \frac{\ln k}{k\sqrt{k}} \qquad (ii) \quad \sum_{k=1}^{\infty} \frac{k^2}{e^k}.$$

2.[20pt.]

(*a*) Use a suitable substitution and a reduction formula to evaluate: $\int xe^{-\sqrt{x}}dx.$

(*b*) Find the area of the surface generated by revolving the parametric curve:

$$x = \cos^2 t, \quad y = \sin^2 t, \quad 0 \le t \le \frac{\pi}{2},$$

about the y-axis.

3.[20pt.] Evaluate the following Integrals:

$$(a) \quad \int_0^{\infty} \frac{dx}{a^2 + bx^2},\ a > 0,\ b > 0. \quad (b) \quad \int \frac{\cos x}{\sin^2 x + 4\sin x - 5}dx.$$

4.[20pt.]

(*a*) Find the total arclength of the cardiod: $r = 1 + \cos\theta$, as θ varies from $\theta = 0$ to $\theta = 2\pi$.

(*b*) Find the area of the region bounded by one loop of the graph of the polar equation: $r = 2\sin 2\theta$.

5.[20pt.]

(*a*) Use L'Hôpital's rule to help evaluate the improper integral:

$$\int_1^{\infty} \frac{\ln x}{x^2} dx.$$

(*b*) Make the u-substitution $u = \sqrt{x}$ in the improper integral:

$$\int_{12}^{\infty} \frac{dx}{\sqrt{x}(x+4)},$$

and evaluate the resulting definite integral.

6.[20pt.] Use partial fraction decompositions to evaluate the following integrals:

$$(a) \quad \int \frac{2x^2+3x+3}{(x+1)^3} dx. \quad (b) \quad \int \frac{x^5-4x^3+1}{x^3-4x} dx.$$

7.[20pt.]

(*a*) Find the volume of the solid that results when the region above the x-axis and below the graph of the ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \ (a > 0, \ b > 0),$$

is revolved about the x-axis.

(*b*) Let a_n be the average value of the function $f(x) = \frac{1}{x}$ over the interval $[1, n]$. Determine whether the sequence $\{a_n\}$ converges and if so find its limit.

8.[20pt.] Evaluate the integral: $I(x) = \int \frac{x^3}{\sqrt{x^2+3}} dx$ by:

(*a*) using Integration by parts.

(*b*) the substitution $u = \sqrt{x^2+3}$.

9.[20pt.]

(*a*) Find the radius of convergence and the interval of convergence for the power series:

$$\sum_{k=0}^{\infty} \frac{(2x-3)^k}{4^{2k}}.$$

(*b*) Differentiate the Maclaurin series:

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k, \text{ for } -1 < x < 1$$

and use the result to show that: $\sum_{k=1}^{\infty} kx^k = \frac{x}{(1-x)^2}$, for $-1 < x < 1$.

10.[20pt.]

(*a*) The region bounded by the x-axis, the graphs of the equation $y = x^2 + 1$, the lines $x = -1$ and $x = 1$ is revolved about the x-axis. Use the the Disk Method to find the volume of the resulting solid.

(*b*) Use Integration by parts to evaluate: $\int_0^1 \tan^{-1} x dx$.

11.[20pt.]

(*a*) For the convergent infinite series : $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^4}$, how many terms n are needed to guarantee that the partial sum $S_n = \sum_{k=1}^{n} \frac{(-1)^{k+1}}{k^4}$ is within 1×10^{-10} of the actual sum $S = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^4}$?

[Hint: Use the error estimate $|S - S_n| \leq a_{n+1}$, where the nth term $a_n = \frac{1}{k^4}$.]

(*b*) Evaluate the integral: $\int_0^{\pi/2} \frac{4 \sin x}{1+\cos^2 x} dx$.

12.[20pt.]

(*a*) Find the volume of the solid whose base is the region bounded by the curves $y = \sqrt{x}$ and $y = \frac{1}{\sqrt{x}}$, for $1 \leq x \leq 4$ and whose cross-sections perpendicular to the x-axis are squares.

(*b*) Use a suitable substitution to show that: $\int_0^1 \frac{x}{x^4+1} dx = \frac{\pi}{8}$.

13.[20pt.]

(*a*) Show that $k^k \geq k!$.

(*b*) Use the comparison test to show that the series $\sum_{k=1}^{\infty} \frac{1}{k^k}$ converges.

(*c*) Use the root test to show that the series $\sum_{k=1}^{\infty} \frac{1}{k^k}$ converges.

14.[20pt.] Suppose that the sequence $\{a_k\}$ is defined recursively by:

$$a_0 = c, \quad a_{k+1} = \sqrt{a_k}.$$

Assuming that the sequence converges, find its limit if:

(*a*) $c = \frac{1}{2}$. (*b*) $c = \frac{3}{2}$.

15.[20pt.] A ship is at anchor in $80ft$ of water. The anchor weighs $500lb$, and the chain weighs $20lb/ft$. The anchor is to be pulled up as the ship gets under way.

(*a*) How much force must be exerted to lift the anchor as it comes aboard the ship? Write an equation expressing the force in terms of the displacement y of the anchor from the ocean floor.

(*b*) How much work must be done to raise the anchor $80ft$ from the ocean floor to the point where it comes aboard the ship?