**HOWARD UNIVERSITY**
**DEPARTMENT OF MATHEMATICS**
**DEPARTMENTAL FINAL EXAMINATION.**
**MATH:158: CALCULUS [III]**

**TUESDAY, DECEMBER 8, 2009: TIME: 4.00PM–6.00PM**
ANSWER ANY TEN [10] PROBLEMS.

**1.[20 Points]**
(a) Find the directional derivative of the function:

$$f(x,y) = x^2 e^{-2y}$$

at the point $P(2, 0)$ in the direction of the vector from $P(2, 0)$ to $Q(-3, 1)$.
(b) Find the maximum rate of increase of $f(x,y)$ at $P(2, 0)$.

**2. [20 Points]**
Use Chain Rule to find $\frac{\partial w}{\partial z}$ if:
$w = r^2 + sv + t^3$, with $r = x^2 + y^2 + z^2$, $s = xyz$, $v = xe^y, t = yz^2$.

**3. [20 Points]**
Solve the vector-initial value problem for $\bar{r}(t)$ given that:
$\bar{r}''(t) = \bar{i} + e^t \bar{j}$, $\bar{r}(0) = 2\bar{i}$, $\bar{r}'(0) = 2\bar{j}$.

**4. [20 Points]**
Show that the line $l: x = -1 + t$, $y = 3 + 2t$, $z = -7$ and the plane $2x - 2y - 2z + 4 = 0$ are parallel, and find the distance $D$ between them.

**5.[20 Points]**
(a) A Wagon is pulled along a level ground by exerting a force of 20*lb* on a handle that makes an angle of $30^\circ$ with the horizontal. Find the work done in pulling the Wagon 100*ft*.

**6. [20 Points]**
Find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ if $z = f(x,y)$ is a differentiable function determined implicitly by the equation: $2xz^3 - 3yz^2 + x^2y^2 + 4z = 0$.

**7. [20 Points]**
Find equations for the tangent plane and the normal line to the graph of the equation:

$$F(x,y,z) = xy + 2yz - xz^2 + 70 = 0,$$

at the point $P(-5, 5, 1)$.

**8. [20 Points]**

Let $f: R^2 \to R$ be the function of two variables defined by:

$$f(x,y) = 0.5x^2 + 2xy - 0.5y^2 + x - 8y.$$

Find the local extrema of the function *f* or saddle point.

**9. [20 Points]**

Find the volume of the largest rectangular box with faces parallel to the coordinate planes that can be inscribed in the ellipsoid:

$$16x^2 + 4y^2 + 9z^2 = 144.$$

**10. [20 Points]**

A particle *P* travels along a smooth curve *C* given by the position vector:

$$\bar{r}(t) = \sqrt{2}t\bar{i} + t^2\bar{j} + t\bar{k}.$$

Find the scalar and vector, tangential and normal components of the acceleration and the curvature $\kappa(t)$ of the path *C* at time $t = 1$

[***Hint***: Scalar components of acceleration formulae are:

$$a_T = \frac{\bar{v}\bar{a}}{\|\bar{v}\|},\ a_N = \frac{\|\bar{v}\times\bar{a}\|}{\|\bar{v}\|}, \kappa = \frac{\|\bar{v}\times\bar{a}\|}{\|\bar{v}\|^3}].$$

**11. [20 Points]**

Evaluate the double integral $\iint_R (2xy - x^2)dA$, where *R* is the rectangle bounded by $-1 \le x \le 2$ and $0 \le y \le 4$.

**12. [20 Points]**

Let $\bar{F}(x,y,z) = xy^2z^4i + (2x^2y + z)\bar{j} + y^3z^2\bar{k}$.

Find the $curl\bar{F} = \bar{\nabla}\times\bar{F}$ and the $div\bar{F} = \bar{\nabla}\bar{F}$

**13. [20 Points]**

Reverse the order of integration and evaluate the resulting integral:

$$\int_0^3\int_{y^2}^9 ye^{-x}dxdy$$

**14. [20 Points]**

Set up a surface integral to show that the unit sphere $x^2 + y^2 + z^2 = 1$ has surface area $4\pi$;

Set up a triple integral to show that the unit sphere $x^2 + y^2 + z^2 = 1$ has volume $4\pi/3$.

**15. [20 Points]**

Evaluate the line integral $\int_C (x+3y)dx + (x-y)dy$ along the curve $C: x = 2\cos t, y = 6\sin t,\ 0 \le t \le \pi/6$.