Calculus III Final Examination Fall 2010 Show all details to earn full credit

Problems 1 through 11, 12 points each; problems 12 through 15, 17 points each

1. Find the angle between $\mathbf{u} = \langle -2, 0, 2 \rangle$ and $\mathbf{v} = \langle 1, 1, 0 \rangle$.

2. Let $P = (1, -1, 1), Q = (1, 2, 1)$, and $R = (-1, 0, 1)$. Find the area of the triangle with vertices P, Q, and R.

3. Find arc length of the parametric curve C given by $\langle t^3, t, \frac{\sqrt{6}}{2} t^2 \rangle, 1 \le t \le 3$.

4. Find an equation of the tangent plane to the graph of $z = x^2 y + x y^3$ at the point $(-1, 1, 0)$.

5. Locate all relative extrema and saddle points of the function $f(x, y) = x^3 + y^3 - 3xy$.

6. Let $f(x, y, z) = y e^{x+z} + z e^{y-x}$. At the point $(2, 2, -2)$, find the unit vector pointing in the direction of most rapid increase of f.

7. Find a nonzero vector $\mathbf{u}$ such that $\mathbf{u} \times \mathbf{u} = \mathbf{u}$. If not possible, explain.

8. Build up a triple integral representing the volume of the unit sphere $x^2 + y^2 + z^2 = 1$.

9. Find the distance between the parallel planes $x + y + z = 1$ and $x + y + z = 3$.

10. Find the directional derivative of $f(x, y) = e^{x^2 y^2}$ at $P = (1, -1)$ in the direction toward $Q = (2, 3)$.

11. Use Lagrange multipliers to minimize $f(x, y) = 3x^2 + y^2$ subject to $xy = 1$. (You may use methods in Calculus I.)

12. Compute $\displaystyle\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \frac{1}{\sqrt{x^2 + y^2}}\, dx\, dy$. (Hint: convert to polar coordinates.)

13. Let C be the triangle with vertices $(0, 0), (1, 0), (1, 1)$ traversed in the counterclockwise direction. Evaluate $\int_C (\sin^3 x + 2y)\, dx + (x^2 y + \cos^3 y)\, dy$. (Green's theorem is helpful.)

14. Compute the double integral $\displaystyle\iint_D e^{x/y}\, dA$, where $D = \{(x, y)| \quad 1 \le y \le 2, y \le x \le y^3\}$

15. Among four functions and three equations defined below:

    (A) $z = 3^{(x+y)} \tan(x + 2y)$ (B) $z = \log_3(1 + (x - y)^2)$, (C) $z = e^{-x} \sin y$ (D) $z = (\cos x)\, e^y$

    (1) $\dfrac{\partial z}{\partial x} = \dfrac{\partial^2 z}{\partial y^2}$ (2) $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$ (3) $\dfrac{\partial^2 z}{\partial x^2} = \dfrac{\partial^2 z}{\partial y^2}$,

    which satisfies equation (1)? which satisfies equation (2)?
    which satisfies equation (3)? which satisfies none? (Show work to support your answers.)