Math 158 Final exam Calculus 3 5/1/02

Please provide step by step solutions with explanations for each step.
Total 200 points; Time limit 2 hrs.

Part I : 15 points each, do all the problems

1.Determine whether the line whose equation is given by $\mathbf{r}(t) = \mathbf{i} + t(2\mathbf{i} + \mathbf{k})$ is parallel or perpendicular or neither to the plane given by $x + 3y - 2z = 5$.

2. A particle is travelling along a curve whose parametric equation is given by $(1+3t^2)\mathbf{i}+4t^2\mathbf{j}+2\mathbf{k}$ where t is time in seconds. Find the distance travelled (arc-length) from $t = 0$ to $t = 2$, and the velocity and speed at $t = 2$ seconds.

3. Find all first and second partial derivatives of $x^3y^2 - 2cos(xy)$.

4. Find all relative extrema and saddle points of $f(x, y) = x^3 + 3xy + 6y$.

5. From the equation $xe^z + yz = x^2$ that determines z as a function of x and y implicitly, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ using implicit differentiation.

6. Find the equation of the tangent plane and normal line to the surface $z = 2xy - y^2$ at the point (1,2,0).

7. Evaluate $\int_0^4 \int_{\sqrt{x}}^2 sin(\pi y^3)\, dydx$ by first changing the order of integration.

8.

$$\text{Evaluate} \int_0^1 \int_{1+x}^{2x} \int_z^{x+z} x\, dydzdx$$

9. Find the area of the portion of the surface $z = xy$ inside the cylinder $x^2 + y^2 = 1$.

10. Use the transformation $u = x/3, v = y/4$ to evaluate the area inside the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$ by integrating over the disk $u^2 + v^2 \leq 1$.

Part II. Answer any two. Each carries 25 points

1. (a) (15 points) Find the work done by the force $\mathbf{F}$ given by $\mathbf{F}(x, y) = y^2\mathbf{i} - 2x^2\mathbf{j}$ acting on a particle that moves along the circle $x = cost, y = sint$ from (1,0) to (0,1). [You may use: $\int_0^{\pi/2} cos^3t = \int_0^{\pi/2} sin^3t = 2/3$.]

(b) (10 points) Find the divergence and the curl of the vector field $\mathbf{F}$ given by $\mathbf{F}(x, y, z) = cosx\mathbf{i} + sinx\mathbf{j} + ln(xy)\mathbf{k}$.

2. Find the volume of the solid enclosed by the cylinder $x^2 + y^2 = 4$ and the surface $z^2 - x^2 - y^2 = 4$.

3. Find the point on the plane $x+3y+z = 1$ within the first octant $(x \geq 0, y \geq 0, z \geq 0)$ that is closest to the origin.

4. Suppose a particle is moving with constant velocity. i.e, $\mathbf{r}'(t) = \mathbf{c}$ for some fixed vector $\mathbf{c}$. Let the magnitude of $\mathbf{c} = c$. Show that $\frac{d^2}{dt^2}\left(||\mathbf{r}(t)||^2\right) = \frac{d^2}{dt^2}\left(\mathbf{r}(t)\cdot\mathbf{r}(t)\right) = 2c^2$ and $\frac{d}{dt}\left(\mathbf{r}(t)\times\mathbf{r}'(t)\right) = 0$.