HOWARD UNIVERSITY
Differential Equations – Math 159
Final Examination
Tuesday, December 8, 2009

Answer any 8 problems and problems 1 and 13 (Mandatory)
Each problem is worth 20 points. To earn the full grade you must show your work

(1) (20pts) (**Mandatory**) The roots of the characteristic equation to a certain linear differential equation with constant coefficients are:

$$10, -10, 10, -3, -2\pm 3i, -2\pm 3i, -2\pm 3i, -3, 10, -2\pm 3i.$$

Write the general solution to that differential equation.

(2) (20pts) Solve each differential equation as specified
   (a) Find the general solution to: $y' + (1+\sin x)y = 0$.
   (b) Find the general solution to:

$$(1+ye^{xy})dx + (2y+xe^{xy})dy = 0.$$

(3) (20pts) Choose an appropriate form for a particular solution of

$$y^{(6)} + 3y^{(5)} + 3y^{(4)} + y^{(3)} = t + 2te^{-t} + \sin t.$$

(4) (20pts) Choose an appropriate form for a particular solution to each of the following differential equations.
   (a) $y'' + 4y = t\cos 2t$.
   (b) $y'' + 4y = 2t^3 + 2\cos 2t + e^{2t}$.

(5) (20pts) Solve the initial value problem

$$y'' - y' - 2y = -5e^{-3t},\ y(0) = 1,\ y'(0) = -2.$$

(6) (20pts) Consider the following Euler equation given by

$$x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 12y = 0.$$

   (a) Use the change of variable $t = \ln x$ to write this equation as a constant coefficient equation in y and t.
   (b) Solve this new equation; and
   (c) Obtain the general solution of the given Euler equation.

(7) (20pts) The growth of a population of about 100,000 bacteria in a culture is modeled by the differential equation

$$\frac{dP}{dt} = kp,$$

Where k is the population growth rate and $P(t)$ is the size of the population after t days. Suppose that 2 days later the population has grown to about 150,000 bacteria.
   (a) Determine the growth rate k.
   (b) Estimate the bacteria population after 7 days.
   (c) How long approximately will it take for the population to triple?

(8) (20pts) Use the method of variation of parameters to find a particular solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 8y = 3e^{-2x}.$$

(9) (20pts) Consider the linear system $\frac{dY}{dt} = AY$, where $Y = \begin{pmatrix} x \\ y \end{pmatrix}$, and A is the 2 x 2 matrix given by

$$A = \begin{pmatrix} 3 & 4 \\ 3 & 2 \end{pmatrix}.$$

(a) Compute its eigenvalues and eigenvectors.
(b) Determine its general solution.
(c) Determine the solution satisfying $Y(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

(10) (20pts) Find a power series solution to the equation

$$(x+8)y' + 4y = 0,$$

then identify the series solution in terms of familiar elementary functions.

(11) (20pts) Solve the general solution to

$$y^{(4)} + 16y = 0.$$

(12) (20pts) Use the Laplace transform to solve the initial value problem

$$y'' + 4y = 3\cos t, \ \ y(0) = 0, \ \ y'(0) = 1.$$

(13) (20pts) **(Mandatory)**
(a) Find the solution to the initial-value problem:

$$y' + te^{y} = e^{y}\sin t, \quad y(0) = 0.$$

(b) Find the solution to the initial-value problem:

$$y' + 3y = 6x + 4, \quad y(0) = 3.$$

(14) (20pts) Find the inverse Laplace transform of the function:
(a) $F(s) = \frac{1}{(s-1)^2(s^2-2s+10)}$; and
(b) $F(s) = \frac{1}{(s-1)(s^2+9)}$.

(15) (20pts) Determine the general form of the fuction $M(x,y)$ and $N(x,y)$ that will make the given differential exact
(a) $(M(x,y)dx + (xe^{y} + x + 2y)dy = 0$
(b) $(e^{y^2} + 2xy - 1)dx + N(x,y)dy = 0.$

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