Howard University

MATH 159 – Differential Equations

Final Exam

December 8, 2015

Solve any 10 of the following problems. Each problem is worth 20 points.
*In order to earn full grade, you **must** show your work.*

1. a) Solve $y' = \dfrac{1}{x^2 - 2x + 1}$, $y(0) = -1$. b) Find the six sixth roots of 1 and draw them in the complex plane.

2. Find the general solution of separable ODE $y' = y - y\cos x$.

3. Sketch the graph of the solutions in the plane (t, P) for the ODE $P' = P(P-2)(P+1)$ and write down one particular solution of it (do not attempt to find the general solution). What can you say about $\lim_{t\to\pm\infty} P(t)$ for the solutions with initial condition $0 < P(0) < 2$?

4. Solve the linear homogeneous ODE $x'' + 4x' + 4x = 0$, $x(0) = 1$, $x'(0) = 0$.

4. Find the general solution of the linear ODE $x'' + 4x = e^{2t}$.

5. Solve $x'' - x = 2$, $x(0) = 0$, $x'(0) = 0$ using the Laplace transform technique.

6. Use Euler's method to evaluate $x(1)$ for the ODE $x' = 2t - x$, $x(0) = 3$, with $h = 0.5$ and compare it with the exact solution.

7. Consider the system $v' = Av$, with $A = \begin{pmatrix} 1 & -\alpha \\ \alpha & 3 \end{pmatrix}$, and determine which kind of equilibrium points you get depending on the value of α.

8. Evaluate from the definition the Laplace transform $F(s)$ of the function $f(t)$ equal to 1 from 0 to 1 and to e^t from 2 to 3.

9. Solve the ODE $x'' - 2x = 4\delta(t-4)$, $x(0) = 0$, $x'(0) = 0$.

10. Find all equilibrium points of the non-linear system

$$\begin{cases} x' = y - yx^2 \\ y' = 2x - y + xy \end{cases}$$

11. Find the general solution of the 2x2 system

$$\begin{cases} x' = 2x - y \\ y' = 4x - 3y \end{cases}$$

and discuss the stability of the equilibrium point.

12. Study the equilibrium point $(-1, -1)$ of the non-linear equation

$$\begin{cases} x' = y - yx^2 \\ y' = 2x - y + xy \end{cases}$$

What happens to that equilibrium point if we add a term ϵx to the second equation?

13. Evaluate e^{At}, where $A = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$.

14. Write an ODE which is solved by both functions e^{2t} and e^{3t}.