HOWARD UNIVERSITY

Differential Equations – Math 159

Final Examination

Tuesday, December 7, 2010

Answer any 8 problems and problems 1 and 13 (Mandatory)

Each problem is worth 20 points. To earn the full grade you must show your work

(1) (20pts) **(Mandatory)** The roots of the characteristic equation to a certain linear differential equation with constant coefficients are: -3; -3; $3\pm 2i$; $3\pm 2i$; $3\pm 2i$; -3; -3; $3\pm 2i$; 2; 2; 2. Write the general solution to that differential equation.

(2) (20pts) Use power series method to solve

$$y^{(4)}=0,\ \ y(0)=0,\quad y'(0)=0,\quad y''(0)=0,\quad y'''(0)=1.$$

(3) (20pts) Find the general solution to the differential equation

$$\left(e^{2y}-y\cos xy\right)dx+\left(2xe^{2y}-x\cos xy+2y\right)dy=0.$$

(4) (20pts) Choose an appropriate form for a particular solution of (do not compute the coefficients)

$$y'''+3y''-4y=5+2te^{-2t}+7e^{t}.$$

(5) (20pts) Find the general solution to

$$y'''+y''=e^{t}\cos t.$$

(6) (20pts) Solve the initial value problem

$$y''+4y'+7y=0,\ \ y(0)=1,\ \ y'(0)=-2.$$

(7) (20pts) Solve

$$y'-5y=-\frac{5}{2}xy^{3}.$$

(8) (20pts) Solve by any method

$$y''+16y=2\cos 4x.$$

(9) (20pts) Consider the linear system $\frac{dY}{dt}=AY$, where $Y=\begin{pmatrix} x \\ y \end{pmatrix}$, and A is the 2 x 2 matrix given by

$$A=\begin{pmatrix} 4 & -3 \\ 6 & -7 \end{pmatrix}.$$

(a) Compute its eigenvalues and eigenvectors.

(b) Determine its general solution.

(c) Determine the solution satisfying $Y(0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

(10) (20pts) Find a power series solution to the equation

$$xy'' - y' = 0,$$

then identify the series solution in terms of familiar elementary functions.

(11) (20pts) Find the general solution to

$$y^{(4)} + 2y'' + y = 0.$$

(12) (20pts) Use the Laplace transform to solve the initial value problem

$$y'' - y' - 6y = 0, \ \ y(0) = 2, \ \ y'(0) = -1.$$

(13) (20pts) **(Mandatory)**

(a) Find the solution to the initial-value problem:

$$y' - 2(\sqrt{y+1})\cos x = 0, \quad y(\pi) = 0.$$

(b) Find the general solution to

$$y' - \frac{y}{x} = xe^x.$$

(14) (20pts) Consider the linear system $\frac{dY}{dt} = AY$, where $Y = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$, and A is the 3 x 3 matrix given by

$$A = \begin{pmatrix} -4 & 1 & 1 \\ 1 & 5 & -1 \\ 0 & 1 & -3 \end{pmatrix}.$$

(a) Compute its eigenvalues and eigenvectors.

(b) Determine its general solution.

(c) Determine the solution satisfying $Y(0) = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$.

(15) (20pts) Find the inverse Laplace transform of the function:

(a) $F(s) = \dfrac{1}{(s^2+4)(s^2-s-6)}$

(b) $F(s) = \dfrac{1}{(s^2+4)(s^2+9)}$.