DIFFERENTIAL EQUATIONS: MATH 159
SPRING 2006
FINAL EXAMINATION
MAXIMUM 200 POINTS

PART A: ANSWER ANY FIVE PROBLEMS.
EACH PROBLEM IS WORTH 24 POINTS.

1. Solve initial value linear problem

$$t\,y' + 2\,y = t^2 - t + 1,\ t > 0,\ y(1) = \frac{1}{2}$$

(Notation: $y' = (d/dt)y$)

2. Find the general solution of the differential equation

$$y' = \frac{y(t-y)}{t^2},\ t > 0.$$

(Hint: Substitute $y = u\,t$ and solve the resulting equation to get the solution.)

3. For the one-parameter family of differential equations

$$y' = y^2 - a\,y + 1$$

find the bifurcation value of the parameter a. Draw the phase lines corresponding to the bifurcation value of a and for values slightly less than and slightly greater than the bifurcation value of a.

4. Find the general solution of the linear system

$$x' = x - 3y$$
$$y' = x + 5y$$

Sketch the phase portrait.

5. Find the solution of the initial value problem

$$x' = 3\,x + 4\,y$$
$$y' = -2\,x - y$$
$$x(0) = 1,\ y(0) = 0$$

Sketch the phase portrait showing this solution.

6. Find the general solution of the system

$$x' = -2\,x - y$$
$$y' = x - 4\,y$$

Sketch the phase portrait.

7, Solve the initial value problem

$$y'' + 2\,y' + 2\,y = e^{2t},\ y(0) = 0,\ y'(0) = 0$$

by the method of undetermined coefficients or by the method of Laplace trasforms.

PART B: ANSWER ANY TWO PROBLEMS.
EACH PROBLEM IS WORTH 40 POINTS.

8 (a) Compute the solution of

$$y'' + 4y' + 20y = -3\sin(2t),\ y(0) = 0,\ y'(0) = 0$$

(b) Compute the amplitude and phase angle of the forced oscillations.

9 (a) Solve the initial value problem

$$y'' + y = 0.5\cos(0.8t),\ y(0) = 0,\ y'(0) = 0$$

(b) Determine the frequency of the beats and the frequency of the rapid oscillations.
(c) Draw the solution curve and the amplitude modulation curve.

10. Solve the initial value problem

$$y'' + 3y = g(t),\ y(0) = 2,\ y'(0) = 0,$$

where

$$g(t) = \begin{cases} t & \text{if } 0 \le t < 1 \\ 1 & \text{if } t \ge 1 \end{cases}$$

11 (a) Find the Laplace inverse of

$$g(s) = \frac{1}{(s-1)(s^2+4)}$$

(b)Use Laplace transform to solve the initial value problem

$$y'' + 4y = 2\cos(3t),\ y(0) = 1,\ y'(0) = 0.$$