1. Find dy/dx for the following functions.

a. $2x+xy^2=y^3$

b. $y=\int_2^{x^4}\frac{t}{\sqrt{t^3+2}}dt$

c. $y=\tan(\cos(5x))$

2. Find the following antiderivatives.

a. $\int \sin^4 x\cos^3 x dx$

b. $\int x^3 e^{-x^2} dx$

c. $\int \frac{x^3+2x-1}{x^2}dx$

3. State and prove the Intermediate Value Theorem. Any theorem(s) used in proving the Intermediate Value Theorem must be stated clearly.

4. Suppose $f:S\to\Re$ for $S\subseteq\Re$ is continuous. For each of the following statements, state whether it is true or false. If it is false, give a counter example.

a) If *S* is bounded then *f(S)* is bounded.

b) If *S* is compact the *f(S)* is compact.

c) If *S* is open, then *f(S)* is open.

5.

(a) Determine if the series converges or diverges: $\sum_{n=1}^{\infty} ne^{-n^2}$

(b) Determine if the series converges or diverges: $\sum_{n=1}^{\infty}\frac{n^n}{n!}$

(c) By reversing the order of integration, evaluate the double integral: $\int_0^4\int_{\sqrt{y}}^2 y\cos x^5 dxdy$

6. Let A be a symmetric matrix ($A^T=A$) with eigenvectors v_1 and v_2 and corresponding distinct eigenvalues λ_1 and λ_2. Show that v_1 and v_2 are orthogonal ($v_1\cdot v_2=0$).

7. Find the eigenvalues and corresponding eigenvectors for $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$.

8. Show by induction that for positive integers n,

$$\begin{bmatrix} \cos\theta & k\sin\theta \\ -\frac{1}{k}\sin\theta & \cos\theta \end{bmatrix}^n = \begin{bmatrix} \cos n\theta & k\sin n\theta \\ -\frac{1}{k}\sin n\theta & \cos n\theta \end{bmatrix},$$

where k is a non-zero number.

9. Using the definition, show that $\lim_{n\to\infty} \frac{n}{n+1} = 1$.

10. Give examples of the following:

a) Nonconvergent sequence and a subsequence which converges.

b) A bounded sequence which does not converge.

c) An increasing sequence which does not converge.