Q1)

(a) Use implicit differentiation to find $\frac{dy}{dx}$ for $x^2+y^3-2y=3$. Then find the equation of the tangent line to the curve at $(2,1)$.

(b) If $y=\int_1^{x^2}\cos t\,dt$, find dy/dx.

(c) Find $\lim_{x\to 0}\frac{1-\cos x}{x^2}$.

Q2) Find the following antiderivatives:

(i) $\int x\sqrt{(2-x)}\,dx$

(ii) $\int t^2\sin t\,dt$

(iii) $\int \sin^3 x\cos^4 x\,dx$

Q3)

(a) Let $f(x)=\frac{1}{x+1}$, for $|x|<1$. Find the power series for $f(x)$.

(b) Using the results from part (a), deduce the power series for $\ln(1+x)$.

Q4)

(a) Define what is meant for a sequence of functions $\{f_n\}$ to converge uniformly on an interval [a , b] to a function f.

(b) Show that the sequence of functions $\{f_n\}$ where $f_n(x)=x^n$ is uniformly convergent on [0 , k] where k<1.

(c) From part (b) above, what conclusion can you deduce if the interval under consideration is [0 , 1].

Q5) Determine if the following series converges or diverges:

(a) $\sum_1^{\infty}\frac{(2n)!}{n!n!}$ (b) $\sum_1^{\infty}\frac{1}{n^3+5n+3}$ (c) $\sum_{n=1}^{\infty}\frac{\ln n}{n}$

Q6) Find the eigenvalues and corresponding eigenvectors for: $A=\begin{bmatrix}1 & 1\\ -2 & 4\end{bmatrix}$.

Q7)

(a) Let u be a vector in R^n such that $u^T u = 1$. Show that the $(n \times n)$ matrix $P = uu^T$ is an idempotent matrix.

(b) Let Q be an idempotent matrix. Verify that $I - Q$ is idempotent and show that $(I-2Q)^{-1} = (I-2Q)$.

Q8) By reversing the order of integration, evaluate the double integral: $\int_0^1 \int_y^1 \frac{\sin(x)}{x} dxdy$

Q9) Find the radius of convergence and interval of convergence for the series: $\sum_{n=0}^{\infty} \frac{n(x+2)^n}{3^{n+1}}$

Q10)

(a) State the Heine-Borel Theorem.

(b) State Bolzano-Weierstrass Theorem

(c) Use the definition of a limit to show that $\lim_{n\to\infty} \frac{3+2\sqrt{n}}{\sqrt{n}} = 2$.