Howard University
Department of Mathematics

Fall 2012 Semester — Saturday, October 27th, 2012.

Senior Comprehensive Mathematics Exam

Name:______________________________

(Please **PRINT** your name)

Signature:______________________________________

Email address: __________________________________

I.D. # __________________________ Tel #: ___________________________

Address: ____

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Ethnicity: (Please circle one)
B Black
W White
H Hispanic
N Native American Indian / Alaskan Native
A Asian / Pacific Islander

Gender: M / F

- *Show all work otherwise NO POINTS WILL BE AWARDED.*
- *All work must be neat and legible OTHERWISE POINTS WILL NOT BE AWARDED.*
- *Partial credits will be given for work which demonstrates a working knowledge of the concepts.*
- *Answer all questions. Each question is worth 10 points.*
- *Use the blue book provided for you answers.*

Do not write in the columns below.

Question 1			Question 7	
Question 2			Question 8	
Question 3			Question 9	
Question 4			Question 10	
Question 5			Exam Total	
Question 6			Exam Grade (Pass or Fail)	

Q1)

(a) Use implicit differentiation to find $\frac{dy}{dx}$ for $x^2 + y^3 - 2y = 3$. Then find the equation of the tangent line to the curve at $(2,1)$.

(b) If $y = \int_1^{x^2} \cos t\,dt$, find *dy/dx*.

(c) Find $\lim_{x \to 0} \frac{1-\cos x}{x^2}$.

Q2) Find the following antiderivatives:

(i) $\int x\sqrt{(2-x)}dx$

(ii) $\int t^2 \sin t dt$

(iii) $\int \sin^3 x \cos^4 x dx$

Q3)

(a) State the Mean Value Theorem.

(b) Find a value of c satisfying the conclusion of the Mean Value Theorem for $f(x) = x^3 - x^2 - x + 1$ on the interval [0 , 2]

(c) Give an example of a function which is continuous at a value x but is not differentiable at x.

Q4)

(a) Define what is meant for a sequence of functions $\{f_n\}$ to converge uniformly on an interval [a , b] to a function f.

(b) Show that the sequence of functions $\{f_n\}$ where $f_n(x) = x^n$ is uniformly convergent on [0 , k] where k<1.

(c) From part (b) above, what conclusion can you deduce if the interval under consideration is [0 , 1].

Q5) Determine if the following series converges or diverges:

(a) $\sum_1^{\infty} \frac{(2n)!}{n!n!}$ (b) $\sum_1^{\infty} \frac{1}{n^3 + 5n + 3}$ (c) $\sum_1^{\infty} (-1)^n n^2 \left(\frac{2}{3}\right)^n$

Q6) Find the eigenvalues and corresponding eigenvectors for: $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$.

Q7) Find the algebraic expression for the null space and range for the matrix $A = \begin{bmatrix} -1 & 3 \\ 2 & -6 \end{bmatrix}$.

Q8) Evaluate the double integral: $\iint_R xy\,dA$ where R is the standard region bounded by the lines and curves:

$x = 0$, $x = 1$, $y = x^2$, and $y = 1 + x$.

Q9) Find the radius and interval of convergence for: $\sum_{n=1}^{\infty} \left(\frac{n+1}{2^n} \right) x^n$.

Q10)

(a) Define what it means to say a sequence $\{a_n\}$ is convergent to L.

(b) Define what it means to say a sequence $\{a_n\}$ is bounded.

(c) Prove or disprove: Every convergent sequence is bounded

(d) Prove or disprove: Every bounded sequence is convergent.