HOWARD UNIVERSITY
DEPARTMENT OF MATHEMATICS
SENIOR COMPREHENSIVE EXAMINATION
NOVEMBER 2, 2013

Name: ______________________________

Id. Number: ______________________________

Email address: ______________________________

Address: ______________________________________

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Signature: ______________________________

⇒ This exam consists of 10 questions. Answer all the questions. Each question is worth 10 points.

⇒ Show all your work as neatly and legibly as possible on the Bluebook provided. **No work, no credit.**

⇒ Good Luck!

Question	**Points**	**Out of**
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
8		10
9		10
10		10
Total		100
GRADE (P or F)		

10 points 1. Evaluate the following limits:

(a) $\lim_{x \to 0^+} \dfrac{\tan x - x}{x - \sin x}$

(b) $\lim_{x \to 0} \left(\dfrac{1}{x} - \dfrac{1}{\sin x} \right)$

(c) $\lim_{x \to \infty} x(\sqrt{x^2 + 4} - x)$

10 points 2. Evaluate the following integrals:

(a) $\int \sec^3 x \, dx$

(b) $\int \dfrac{e^t}{e^{2t} + 3e^t + 2} \, dt$

(c) $\int x \sin^{-1} x \, dx$

(d) $\int_0^\pi \int_x^\pi \dfrac{\sin y}{y} \, dy \, dx$ (Hint: Reverse the order of integration first)

10 points 3. (a) State the Intermediate Value Theorem.

(b) Let $f(x)$ be a continuous function from $[0, 1]$ onto $[0, 1]$. Prove that there exists a c in $[0, 1]$ such that $f(c) = c$. (Hint: Use the Intermediate Value Theorem)

10 points 4. For the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 6 \end{bmatrix}$,

(a) Find all the eigenvalues of the matrix A.

(b) Find all the eigenvectors of the matrix A.

(c) Find the null space of A.

10 points 5. (a) Define what it means to say that the infinite series $\sum_{n=1}^{\infty} a_n$ converges.

(b) Determine if the series $\sum_{n=1}^{\infty} (-1)^n \dfrac{1}{\sqrt{n} + \ln n}$ converges or not.

(c) Determine if the series $\sum_{n=1}^{\infty} \dfrac{n!}{n^n}$ converges or not.

10 points 6. (a) Define what it means to say a set of vectors is linearly dependent.

(b) Let V be a set of vectors containing the zero vector. Is V linearly independent or dependent? Justify your answer.

10 points 7. Let $I = \int_0^1 x \ln x \, dx$. Is I an improper integral? In either case evaluate I.

10 points 8. Set up a double or triple integral to represent the volume of the sphere $x^2 + y^2 + z^2 = 1$ and show the details to reach the answer $\frac{4\pi}{3}$.

10 points 9. For the sequence of functions $g_n(x) = \frac{1}{n} e^{-nx}$,

(a) Find the pointwise limit of the sequence.

(b) Show that the sequence converges uniformly on $[0, \infty)$.

10 points 10. (a) Give the definition of a Cauchy sequence.

(b) Prove that every convergent sequence is a cauchy sequence.

(c) Prove that every convergent sequence is bounded. Is the converse true? Prove or disprove.