HOWARD UNIVERSITY
DEPARTMENT OF MATHEMATICS
SENIOR COMPREHENSIVE EXAMINATION
OCTOBER 25, 2014

Name: ______________________________

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Signature: ______________________________

⇒ This exam consists of 10 questions. Answer all the questions in increasing numerical order in the provided bluebook. Each question is worth 10 points.

⇒ Show all your work as neatly and legibly as possible. Make your reasoning clear. In problems with multiple parts, be sure to go on to subsequent parts even if there is some part you cannot do.

Question	**Points**	**Out of**
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total		100
GRADE (P or F)		

10 points 1. Evaluate the following limits OR state if it does not exist. Justify your reasoning.

(a) $\lim_{x\to 0} \frac{5-5\cos x}{e^x - x - 1}$

(b) $\lim_{x\to 1} \left(\frac{x}{x-1} - \frac{1}{\ln x}\right)$

(c) $\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+y^2}$

10 points 2. (a) Find all the points at which the graph of $x^3+y^3-9xy=0$ has a horizontal tangent.

(b) Let $G(x) = \int_1^{e^{2x}} \frac{1}{\sqrt{t}}\, dt$. Find the derivative of G at $x=2$, i.e., find $G'(2)$.

10 points 3. Evaluate the following integrals.

(a) $\int_0^{2\sqrt{\ln 3}} \int_{y/2}^{\sqrt{\ln 3}} e^{x^2}\, dxdy$

(b) $\int \frac{6x+7}{(x+2)^2}\, dx$

(c) $\int_C xydx + (x+y)\, dy$ along the curve $y=x^2$ from $(-1,1)$ to $(2,4)$

10 points 4. Set up a double integral representing the surface area of a sphere of radius a and evaluate the integral.

10 points 5. (a) Determine if the series $\sum_{n=1}^{\infty} \frac{n^2+1}{2n^2+n-1}$ converges.

(b) Determine if the series $\sum_{n=1}^{\infty} \frac{(\sin n)(\ln n)^n}{n^n}$ converges.

(c) Determine the values of x for which the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}(3x-1)^n}{n^2}$ converges absolutely.

10 points 6. (a) Define what it means to say that: *d is a metric on the set X.*

(b) For $\mathbf{x}, \mathbf{y} \in \mathbf{R}^2$, let $d(\mathbf{x}, \mathbf{y}) = \max(|x_1 - y_1|, |x_2 - y_2|)$, where $\mathbf{x} = (x_1, x_2)$, $\mathbf{y} = (y_1, y_2)$. Prove that d is a metric on $\mathbf{R}^2$.

10 points 7. Use mathematical induction to prove that $n! \leq n^n$ for any integer $n \geq 1$.

10 points 8. (a) Give the epsilon-delta definition of a what it means to say: *a function f is continuous at the point $x = a$.*

(b) By ONLY using the epsilon-delta definition, show that the function $f(x) = \dfrac{2x}{x+2}$ is continuous at $x = 2$.

10 points 9. (a) Define what it means to say *the vectors $\mathbf{v_1}, \mathbf{v_2}, \ldots, \mathbf{v_k}$ are linearly dependent.*

(b) Show that in the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 3 & 2 \\ 4 & 9 & 5 \end{bmatrix},$$

the row vectors are linearly dependent.

(c) Find the rank of the matrix A in part (b).

10 points 10. (a) Let U and V be vector spaces over $\mathbf{R}$. Define what it means to say: *a mapping $F : U \to V$ is a linear transformation.*

(b) Suppose $F : \mathbf{R}^3 \to \mathbf{R}^2$ is defined by $F(x_1, x_2, x_3) = (x_1 + x_2 + x_3, 2x_1 - 3x_2 + 4x_3)$.

i. Find the matrix A such that $F(\mathbf{x}) = A\mathbf{x}$ where $\mathbf{x} = (x_1, x_2, x_3)$.

ii. Show that F is a linear transformation.

iii. Describe the kernel of F.

iv. Find the nullity of F.