SENIOR COMPREHENSIVE EXAM

SPRING 2009
April 4. 2009
Please do all problems! Each problem is worth 10 points.

1. Find dy/dx for the following functions.

   a. $2x + xy = y^2$

   b. $y = \dfrac{\ln x}{e^{2x}}$

   c. $y = \int_1^{x^2} \sin(t^2)\,dt$

   d. $y = \tan(\cos(5x))$

2. Find the following antiderivatives.

   a. $\int \sin^3 x \cos^2 x\,dx$

   b. $\int x^2 e^x\,dx$

   c. $\int \dfrac{x^3 + 2x - 1}{x}\,dx$

3. Give an example of a function that is continuous at a point but not differential at that point. Prove your example is continuous but not differential at the point.

4. Determine if the following series converges or diverges.

   a. $\sum_{n=1}^{\infty} \dfrac{\sqrt{x}}{x^2 + 2x - 1}$

   b. $\sum_{n=0}^{\infty} \dfrac{e^n}{n!}$

   c. $\sum \sin\left(\dfrac{\pi n^2}{2n^2 + n - 1}\right)$

5. Prove or disprove that AB = BA, whenever A and B are 2x2 matrices.

6. Find the inverse of $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$

7. State the following theorems or definitions. (Each part is worth 2 points.)

(a) Mean Value Theorem

(b) Group

(c) Heine-Borel Theorem

(d) Bolzano-Weierstrass Theorem

(e) First Isomorphism Theorem for Groups

8. Let $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, where a and b are any fixed real numbers. Use induction to show that

$\forall n \in \mathbb{N}, A^n = \begin{bmatrix} a^n & 0 \\ 0 & b^n \end{bmatrix}$.