1. Find dy/dx for the following functions.

a. $\sin(x+y) = (2x+1)^2 y$

b. $y = \int_2^{x^2} (e^{t^2} + 1)dt$

c. $y = \tan(\cos(5x))$

2. Find the following antiderivatives.

a. $\int \sec^3 x \tan x dx$

b. $\int x^5 e^{x^3} dx$

c. $\int \frac{1}{x^2 - 4} dx$

3. (a) State Mean Value Theorem for derivatives.

(b) Let $f(x) = x^3 - x^2 - x + 1$. Find a value c satisfying the conclusion of the Mean Value Theorem as stated in part (a) above on the interval $[0, 2]$.

4. Give clear description for the following:

a) Give two examples of a function that is continuous on an interval but is not differentiable on the interval.

b) Give an example of a sequence of functions $\{f_n\}$ that converges pointwise but does not converge uniformly.

c) State the condition under which a power series can be differentiated or integrated term by term.

d) When using the integral test for the convergence of a series, what can you say about the value of the integral and the sum of the series?

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(a) Determine if the series converges or diverges: $\sum_{k=0}^{\infty} \frac{(-1)^k k!}{e^k}$

(b) Determine if the series converges or diverges: $\sum_{n=1}^{\infty} ne^{-n^2}$

(c) By reversing the order of integration, evaluate the double integral: $\int_0^4 \int_{\sqrt{x}}^{2} \frac{3}{4+y^3} dydx$

6. Let $T : \Re^3 \to \Re^2$ be the linear transformation given by: $T\left[\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}\right] = \begin{bmatrix} x_1 - x_2 \\ x_2 + x_3 \end{bmatrix}$. Find the matrix A such that $T(\underset{\sim}{x}) = A(\underset{\sim}{x})$. Describe the null space and range of T and find the nullity and rank of T.

7. Find the eigenvalues and corresponding eigenvectors for $A = \begin{bmatrix} 4 & -5 \\ 1 & -2 \end{bmatrix}$.

8. Find the interval of convergence of the series: $\sum_{k=1}^{\infty} \frac{(-1)^k}{k^2 3^k}(x+2)^k$.

9. (a) Give an exact definition of what is meant for a sequence $\{a_n\}$ to converge.

(b) Using the definition from part (a) above, show that $\lim_{n\to\infty} \frac{2n-3}{n+1} = 2$.

10. (a) Give a definition of uniform convergence for a sequence of functions.

(b) Show that the sequence $\{f_n\}$ where $f_n(x) = \frac{1}{x+n}$ is uniformly convergent in $[0, 1]$