Math 157 **Calculus II** **Final Exam** **12/3/2019**

SHOW ALL WORK. Justify your answers! Simplify your answers.
Show details to earn full points. Give exact answers whenever possible.
Each of the 15 problems = 20 pts. Solve any 10 problems below. Exam total = 200 pts.

1. (a) Find the area of the region enclosed by the curves $y = 1 - 2x^2$ and $y = 4x^2 - 1$.
   (b) Find the average value f_{ave} of the function $f(x) = \ln x$ on the interval $[1, e]$.

2. (a) Find volume of rotating region bounded by $y = x^5$, $y = x$, $x = 0$, $x = 1$, about the x-axis.
   (b) Find the resulting volume when the region between $y = e^{-x^2}$ and $y = 0$ and to the right of $x = 0$ is rotated about the y-axis.

3. A circular swimming pool has a diameter of 12 ft, the sides are 3 ft high, and the water is 2 ft deep. Given that water weighs $62.5\,\text{lb}/\text{ft}^3$, do the following.
   (a) Express the work needed to pump all of the water over the side as a limit of a Riemann sum.
   (b) Evaluate the limit in part (a) by expressing it as an integral.

4. Evaluate the following integrals:
   (a) $\displaystyle\int_0^{\pi} (\sin x)^5 (\cos x)^7 \, dx$
   (b) $\displaystyle\int_0^{1} e^{\sqrt{x}} \, dx$ (*Hint:* Let $x = t^2$.)
   (c) $\displaystyle\int_2^{\infty} x^2 e^{-2x} \, dx$
   (d) $\displaystyle\int_{-2}^{2} \frac{1}{\sqrt{2-x}} \, dx$

5. Evaluate the following integrals:
   (a) $\displaystyle\int (\sec x)^{3/2} \tan x \, dx$
   (b) $\displaystyle\int \frac{5}{(x+1)(x^2+4)} \, dx$
   (c) $\displaystyle\int \frac{dx}{x^2 + 2x + 5}$
   (d) $\displaystyle\int \frac{x+2}{\sqrt{x^2+2x-3}} \, dx$

6. Determine if the following series are absolutely convergent, conditionally convergent or divergent:
   (a) $\displaystyle\sum_{n=1}^{\infty} \arctan(n^2+1)$
   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1}$
   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n \cos\sqrt{n}}{n^4+1}$
   (d) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln(n))^4}$

7. Find the sum of the following infinite series.
   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n^2-1}$ (*Hint:* Use partial fractions.)
   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n - 2^n}{4^n}$

(continued on reverse side)

8. Determine if each of the following sequences (a_n) converges, and if so, compute its limit:

   (a) $a_n = \dfrac{\sin n}{n}, \qquad n \geq 1.$ (b) $a_n = \left(\dfrac{n+1}{n}\right)^{4n}, \qquad n \geq 1.$

9. Find the radius of convergence and the interval of convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(4x-1)^n}{n \cdot 3^n}$.

   Note: Be sure to check convergence/divergence at the boundary points.

10. The interval $[1,3]$ is partitioned into n subintervals $[x_{k-1}, x_k]$ for $k = 1, \ldots, n$, of width $\Delta x = x_k - x_{k-1}$. For each $k = 1, \ldots, n$, choose any x_k^* such that $x_{k-1} \leq x_k^* \leq x_k$. Let the function f be continuous on $[1,3]$. Do the following.

    (a) State the limit definition of $\displaystyle\int_1^3 f(x)\,dx$.

    (b) If $n = 4$, write the midpoint approximation for the integral in part (a) in terms of f.

11. (a) Find the length of the curve $y = \ln(\cos x)$ on the interval $0 \leq x \leq \pi/3$.

    (b) Find the area of the surface obtained by rotating the arc of the parabola $y = x^2$ between points $(2,4)$ and $(3,9)$ about the y-axis.

12. How many initial terms of the Maclaurin series for $\sin x$ are required to approximate $\sin 1$ correct to four decimal places? Justify your answer. Find the above approximation.

13. Find the centroid of the region in the first quadrant between the line $x+y=1$ and the circle $x^2+y^2=1$.

14. (a) Find the slope of the tangent line to $r = 2(1+\sin\theta)$ at $\theta = \frac{\pi}{4}$

    (b) Find the area bounded by the curve $r = 2\sqrt{2}(1+\sin\theta)$ in polar coordinates.

15. (a) Convert $(x^2+y^2)^{5/2} = 3(x^2-y^2)$ into polar coordinates.

    (b) Convert $r = 6\cos\theta + 8\sin\theta$ into Cartesian coordinates. What curve in the plane does this equation represent?