Calculus I Final Exam - Offline (in-person) - Version
May 2, 2023
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MUST GIVE STEP BY STEP EXPLANATIONS TO GET CREDIT FOR ANSWERS.
No calculators or electronic devices are permitted.

PART I

Do all three problems. EACH WORTH 24 POINTS.

1. Given the function $f(x) = 4 - 9x - 3x^2 + x^3$.
   (a) Find the intervals of increase or decrease.
   (b) Find the local extreme values.
   (c) Find the intervals of concavity and the inflection points.
   (d) Using the information from (a) to (c) along with its behavior at $\pm\infty$, graph the function f.

2. Find the derivative of the function $f(x)$ using the definition of derivative (LIMIT FORMULA).
$$f(x) = 3x^2 - 4$$

3. Given the function $f(x) = 3x - x^2$ defined on $[0, 1]$ which is partitioned into n subintervals.
   (a) Find the Riemann sum approximation for $f(x)$ over the interval $[0, 1]$ by taking right end points.
   (b) Find the area of the region bounded by the graph of $f(x)$, the $x-$axis and the vertical lines $x = 0$ and $x = 1$ using Riemann Sum approximation.
   (c) Use Fundamental Theorem of Calculus to verify your solution obtained in part (b).

PART II

Choose any 8 problems. EACH WORTH 16 POINTS.

1. Let f be a function defined as follows:
$$f(x) = \begin{cases} 3x - 4, & \text{if } x < 2 \\ 4, & \text{if } x = 2 \\ x, & \text{if } x > 2 \end{cases}$$
   (a) Find $\lim_{x \to 2^-} f(x)$.
   (b) Find $\lim_{x \to 2^+} f(x)$.
   (c) Find $\lim_{x \to 2} f(x)$ if it exists. If it does not exist, explain the reason.

(d) Is f continuous at $x = 2$? Explain the reason to your answer.

2. Find an equation of the line tangent to $y = \dfrac{x+1}{e^{x^2}+1}$ at $\left(0, \frac{1}{2}\right)$.

3. Find the horizontal and vertical asymptotes of th curve $f(x) = \dfrac{4x^2+3x+11}{x^2-2x-8}$ (use limit concept).

4. Find the values of c as a conclusion of the Mean Value Theorem for the function $f(x) = 6x - x^2 - 7$ defined on $[2, 3]$.

5. Determine the possible x-coordinates at which the curve $y^3+2y^2-y^5 = 6x^4-4x^3-6x^2$ could have horizontal tangent lines.

6. Use logarithmic differentiation to find y', where $y = x^{\cos(x)}$.

7. Find the linearization (linear approximation) $L(x)$ of $f(x) = \ln(x^2)$ at $a = 1$ and use it to approximate $\ln((1.2)^2)$.

8. A table of values for f, g, f', and g' is given.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	3	4	6
2	1	8	5	7
3	7	2	7	9

(a) If $h(x) = g(f(x))$. Find $h'(1)$

(b) If $p(x) = f(x)g(x)$. Find $p'(2)$

(c) If $q(x) = f(x)/g(x)$. Find $q'(3)$.

9. Identify the type of indeterminate forms and evaluate the following limits:

$$(a)\ \lim_{x\to 0}\left[\frac{\sin x - x}{x^3}\right] \qquad\qquad (b)\ \lim_{x\to 1^+}(x)^{1/(x-1)}$$

10. The length l of a rectangle is decreasing at the rate of 1 cm/sec while the width w is increasing at the rate of 1 cm/s. When $l = 12$ cm and $w = 5$ cm, find

(a) the rate of change of the area and the length of the diagonals of the rectangle. Also, interpret the rate of change in both the cases.

(b) the dimensions l and w with perimeter 100 m whose area is as large as possible.

11. If $y(x) = \displaystyle\int_x^0 \cos(2t)dt$, evaluate $\dfrac{dy}{dx}$. Also, evaluate $y\left(\dfrac{\pi}{2}\right)$.

12. Evaluate the following integrals using appropriate method:

$$(a)\ \int_1^4 \frac{\sqrt{t}+t}{t}\,dt \qquad (b)\ \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (2t+\cos t)\,dt \qquad (c)\ \int 4x^2\sqrt[3]{1-x^3}dx$$