Math 157 – Calculus II Final Exam – Spring 2022

April 26, 2022

SHOW ALL WORK. Justify your answers! Simplify your answers.
Show details to earn full points. Give EXACT answers whenever possible.
Solve all parts of any 10 out of the 15 problems below.
Each of the 15 problems = 20 pts. Exam total = 200 pts.

1. (a) Find the area of the region enclosed by the curves $y = 4x - 6$ and $y = x^2 - x$.
   (b) Find the average value f_{ave} of the function $f(x) = \sin x$ on the interval $[0, \pi]$.

2. Find the volume of the solid resulting from revolving the region bounded by $y = x$ and $y = x^4$ about:

   (a) the x-axis; (b) the y-axis.

3. Evaluate the integrals:

   (a) $\displaystyle\int (\ln x)^2\, dx$; (b) $\displaystyle\int \frac{\cos x - \sin x}{\cos x + \sin x}\, dx$.

4. Evaluate the integrals:

   (a) $\displaystyle\int_1^{\infty} xe^{-3x}\, dx$; (b) $\displaystyle\int_0^1 \frac{1 + 2x}{1 + x^2}\, dx$.

5. Evaluate the integrals:

   (a) $\displaystyle\int \sin^2(2x)\cos^3(2x)\, dx$; (b) $\displaystyle\int \sec^3 x \tan x\, dx$.

6. Evaluate the integrals:

   (a) $\displaystyle\int \frac{\sqrt{9 - 16x^2}}{x^2}\, dx$; (b) $\displaystyle\int \frac{5}{(x + 2)(x^2 + 1)}\, dx$.

7. Find the exact area of the surface obtained by revolving about the y-axis the section of the curve $y = 1 + x^2$ between the points $(0, 1)$ and $(\sqrt{2}, 3)$.

8. Find the length of the curve defined by $x = 1 - \sin^3 t, y = 1 - \cos^3 t$ over the interval $0 \leq t \leq \frac{\pi}{2}$.

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9. Determine if each sequence converges or diverges, and find the limit if it converges:

   (a) $\left(\dfrac{\cos n}{\ln n}\right)_{n\geq 2}$; (b) $\left(\dfrac{n!}{3^n}\right)_{n\geq 1}$.

10. Determine if each series converges or diverges, and find the limit if it converges:

    (a) $\displaystyle\sum_{n=1}^{\infty} \ln\left(1+\frac{1}{n}\right)$; (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n+(-1)^n}{5^n}$.

11. Determine whether each series converges or diverges using an appropriate test:

    (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos\sqrt{n}}{n^3}$; (c) $\displaystyle\sum_{n=1}^{\infty} \arctan(n^2-1)$;

    (b) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{5n+1}$; (d) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n^3}{n!}$.

12. (a) Sketch the polar curve $r = 4\sin 3\theta$ for $0 \leq \theta \leq \pi$, with an arrow showing the direction as θ increases. Indicate the start and finish point.

    (b) Find the area enclosed by one petal of the curve $r = 4\sin 3\theta$.

13. Find the radius and interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(2x-5)^n}{n\,3^n}$. If the radius is finite and nonzero, determine convergence/divergence at each endpoint.

14. (a) Find an equation of the line tangent to the curve given by $x = 2 + \ln t$, $y = t^2 - 3$ at the point $(2, -2)$.

    (b) Find the vertices, foci, and asymptotes of the hyperbola $\dfrac{x^2}{36} - \dfrac{y^2}{64} = 1$ and draw its graph.

15. A cable 200 meters long weighs 50 Newtons and hangs from the side of a building. Do the following:

    (a) Express the work needed to pull the upper half of the cable to the top of the building as a limit of a Riemann sum.

    (b) Evaluate the limit in part (a) by expressing it as an integral.