HOWARD UNIVERSITY
Final Examination
College Algebra II (MATH-010), Fall 2006

Total points: 200

1. (a) Given $f(x) = \dfrac{4x-3}{2x-4}$, find $f^{-1}(x)$, check your answer, and determine the domain and range of $f(x)$.

   (b) Does every function $g(x)$ have an inverse function $g^{-1}(x)$? If you answer yes, explain why? If not give an example of a function $g(x)$ that has no inverse also indicating why?

2. Solve for x and simplify when possible.

   (a) $9^{\frac{1}{2}x^2-1}+1=4$ (b) $5^{2x-1}=2^{1-x}$ (c) $\log_2(x+4)+\log_2(x-2)=4.$

3) Graph the following functions indicating all intercepts and asymptotes.

   (a) $f(x) = e^{-x} - 1$

   (b) $g(x) = \ln|x|$, that is,

$$g(x) = \begin{cases} \ln x & \text{if } x > 0 \\ \ln(-x) & \text{if } x < 0. \end{cases}$$

4. Determine the type of conic section (hyperbola, parabola, or ellipse) given by

$$4x^2 - y^2 - 24x - 4y + 16 = 0$$

   and write down the coordinates of the center, vertices and foci.

5. Suppose you invest \$1200 in a venture that yields 10% compounded quarterly. How much time will it take for your investment to triple?

6. Let

$$A = \begin{pmatrix} 1 & -6 & 3 \\ 2 & -7 & 3 \\ 4 & -12 & 5 \end{pmatrix}$$

   (a) Compute the product $A \cdot A$ (show your work!)

   (b) Compute the determinant of A

7. Use Cramer's rule to find x and y, given that

$$2x - y = -1$$
$$x + \frac{1}{2}y = \frac{3}{2}$$

8. Solve the system of equations:

$$\begin{aligned} x + 2y + z &= 1 \\ 2x - y + 2z &= 2 \\ 3x + y + 3z &= 3 \end{aligned}$$

9. (a) Graph the region that satisfies the constraints

$$x \geq 0, \quad y \geq 0 \quad 2x + 3y \geq 6 \quad 3x + 2y \geq 6 \quad x + y \leq 10$$

(b) Subject to the constraints in part (a), find the maximum and minimum values of the objective function

$$z = x + y - 1$$

and the coordinates x and y at which these values occur.

10. Find the solution to the nonlinear system:

$$\begin{cases} x^2 + y^2 = 4 \\ x^2 + 2y = 4 \end{cases}$$

11. (a) Find the sum of the first 40 terms of the series given by the general formula $\{3n - 4\}$.

(b) Find a formula to compute the sum of the first n even numbers. and apply it to sum $2 + 4 + \cdots + 20$.

(c) Suppose that $\{a_n\}$ is an arithmetic sequence such that $a_8 = 75$ and $a_{20} = 39$, find the common difference d and express the n^{th} term a_n by a formula that only depends on n.

12. (a) Find the sum of the infinite series $\sum_{n=1}^{\infty} \frac{3^{n+1}}{5^n}$.

(b) Find and simplify the fourth term in the binomial expansion of $(x - 2y)^6$.

13. (a) Find t so that $t + 3$, $2t + 1$, and $5t + 2$ are consecutive terms of an arithmetic sequence.

(b) Write down the n^{th} term of the sequence $\{a_n\}$ suggested by the pattern:

$$1, \frac{1}{2 \cdot 1}, \frac{2 \cdot 1}{3 \cdot 2 \cdot 1}, \frac{3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1}, \cdots$$

in other words, find a simplified general formula for a_n.

14. In how many ways can a committee consisting of 2 faculty members and 3 students be formed from a pool of 6 faculty and 10 students eligible to serve on the committee?