College Algebra II – Math 010   Spring 2016   Final Examination
Howard University   Department of Mathematics

Instructions: *This examination is worth a total of 200 points. Answer all questions. Show your work neatly and coherently in the blue booklet provided.*

(1) (15 pts) Let $P(x) = x^3 - 5x^2 + 2x + 12$.

(a) Show that 3 is a zero of $P(x)$.

(b) Express $P(x)$ as a product of linear factors.

(2) (20 pts) Consider the rational function $f(x) = \dfrac{2x - 8}{x^2 - 9}$

(a) Find the domain of f.
(b) Find the x-intercepts and the y-intercepts of f, if any.
(c) Find the equations of the vertical and horizontal asymptotes, if any.
(d) Find the end behavior of $f(x)$ as x increases, or decreases, without bound.

(3) (15 pts) Solve the rational inequality: $\dfrac{2x - 1}{x + 3} > 0$. Graph the solution on a number line.

(4) (15 pts) Solve by substitution or *any* method of elimination: $\begin{cases} x - y + z = 2 \\ 2x + y + z = 7 \\ 5x + 2y + z = 12 \end{cases}$

(5) (15 pts) Solve ONLY using Cramer's rule: $\begin{cases} 4x + 5y = -3 \\ 6x - 7y = 2 \end{cases}$

(6) (15 pts) Let $A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 3 \\ 1 & -1 & 0 \end{bmatrix}$.

(a) Show directly, *by a step-by-step evaluation of a suitable matrix product*, that A and B are inverses.

(b) Use part (a) to solve the following system of linear equations *by matrix inverse methods*:

$$\begin{cases} 3x_1 - 2x_2 - 4x_3 = 3 \\ 3x_1 - 2x_2 - 5x_3 = 7 \\ -x_1 + x_2 + 2x_3 = 1 \end{cases}$$

(7) (10 pts) Evaluate the determinant by using cofactors: $\begin{vmatrix} -1 & 3 & 5 \\ 2 & 1 & 4 \\ -3 & -2 & 1 \end{vmatrix}$

(8) (20 pts) For a given arithmetic sequence, the nineteenth term is - 69, and the fifth term is 15.

(a) Find the value of the thirtieth term.

(b) Find the sum of the first thirty terms.

(9) (15 pts) For a given geometric sequence, the eleventh term is $\dfrac{31}{9}$, and the fourteenth term is -93. Find the value of the eighteenth term.

(10) (10 pts) How many ways are there of choosing five colors, without replacement, from eight distinct colors, if

    (a) the order of the choices is *not* taken into consideration?

    (b) the order of the choices *is* taken into consideration?

(11) (10 pts) An ordinary (fair) die is a cube with the numbers 1 through 6 on the sides (represented by painted spots). Imagine that such a die is rolled twice in succession and that the face values of the two rolls are added together. This sum is recorded as the outcome of a single trial of a random experiment. Compute the probability that the sum of the die is equal to 9. Write your answer as an exact fraction.

(12) (20 pts) A hyperbola is given by $\dfrac{(x-3)^2}{9} - \dfrac{(y-1)^2}{4} = 1$.

(a) Find the coordinates of its center, vertices, and foci; and find the equation of its asymptotes.

(b) Sketch its graph, labeling the vertices and indicating the asymptotes by a dashed line.

(13) (20 pts) Complete the square to express the equation of the conic given by

$$16x^2 + 4y^2 + 96x - 16y + 96 = 0$$

in standard form. Then identify the type of conic described and sketch its graph.