Department of Mathematics Howard University

Applied Calculus (Math-026)- Final Examination

December 13, 2005

Do any ten (10) problems. Show all work.
The time for this examination is two (2) hours.

1. [20 Points]
Find the derivative $\frac{dy}{dx}$ for each of the following:

a. $y = \frac{2x^2-1}{3-x^2}$

b. $y = x^2\, e^{-2x}$

c. $y = \ln(x^2 - 4)$

d. $y = (2x - 5)^6$

e. $y = \sqrt[4]{\frac{2x-1}{1+2x}}$

2. [20 Points]
Find the following limits, or indicate that the limit does not exist.

a. $\lim_{x\to -1} \frac{x^2-2x-3}{x+1}$

b. $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9}$

c. $\lim_{x\to +\infty} \frac{x^2+4x+5}{3x^2+2x+1}$

3. [20 Points]
a. Find an equation for the line tangent to the graph of $f(x) = 2x^2 + 3$ at the point $(1, 5)$.

b. Let $y = 5x^3 - x^2 + 8x + 1$.
Determine the rate of change of y units with respect to x when x is 4.

4. [20 Points]
A child-care center determines that its total weekly costs, in dollars, of operations are

$$C(x) = 2x^2 + 40x + 1200$$

where x is the number of children enrolled. The center charges tuition of \$200 per week per child, and can enroll any desired number of children.
a. What is the marginal cost?

b. Find the weekly revenue function $R(x)$.

c. Obtain and simplify a formula $P(x)$ for the weekly profit.

d. What enrollment will minimize profit for the center?

5. [20 Points]
When an object is launched vertically upward from ground level with an initial velocity of 72 feet per second, its location after t seconds will be $S(t) = -16t^2 + 72t$ feet above the ground.
a. What is the object's velocity $v(t)$?

b. When does the object stop rising?

c. What is the maximum height attained by the object?

6. [20 Points]
Find the following integrals:

(a) $\int x\sqrt{x^2+4}\,dx$

(b) $\int \frac{4x}{9-4x^2}\,dx$

(c) $\int x^2 e^x\,dx$

(d) $\int x^2 e^{x^3+2}\,dx$

7. [20 Points]
Evaluate the following integrals:

a. $\int_1^4 (\sqrt{x}+2)\,dx$

b. $\int_{-1}^3 \frac{x}{\sqrt{x^2+7}}\,dx$

c. $\int_1^2 \frac{x^4-x^3}{2x^5}\,dx$

8. [20 Points]
Use implicit differentiation to find $\frac{dy}{dx}$:
a. $x^2y^4 = e^{2x}$
b. $5x - x^2y^3 = 2y + 10$

9. [20 Points]
Consider the function $f(x) = x^4 + 8x^3 + 18x^2 - 5$.
a. Determine the intervals where $f(x)$ is increasing and those where $f(x)$ is decreasing.

b. Determine where the graph of $f(x)$ is concave up and where it is concave down.

c. Find the relative extrema and inflection point.

d. Sketch the graph.

10. [20 Points]
For several weeks, the highway department has been recording the speed of freeway traffic flowing past a certain downtown exit. The data suggest that between 1 pm. and 6pm. on a normal weekday, the speed of the traffic at the exit is approximately $S(t) = t^3 - \frac{21}{2}t^2 + 30t + 20$ miles per hour, where t is the number of hours past noon.
At what time between 1pm. and 6pm. is the traffic moving fastest, and at what time is it moving the slowest?

11. [20 Points]

a. Find the particular solution of the given differential equation that satisfies the given condition:

$\frac{dy}{dx} = \frac{x}{y^2}$; $y = 3$ when $x = 2$.

b. Find the general solution of the given differential equation: $\frac{dy}{dx} = 3x^2 + e^{2x} + 5$.

12. [20 Points]

Suppose the total cost in dollars of manufacturing x units of a particular commodity is given by $C(x) = 3x^2 + x + 48$

a. Express the average manufacturing cost per unit as a function of x.

b. For what value of x is the average cost equal to the marginal cost?

13. [20 Points]

For $f(x, y) = 4x^2 - 3x^2y^3 + 5y^2 - 6$, find each of the following partial derivatives:

(a) f_x (b) f_y (c) f_{xx}
(d) f_{yy} (e) f_{xy} (f) f_{yx}

14. [20 Points]

The highway department is planning to build a picnic area for motorist along a major highway. It is to be rectangular with an area of 5,000 square yards and is to be fenced off on three sides not adjacent to the highway. What are the dimensions of the picnic area if all 200 yards of fencing available is used?