Howard University Department of Mathematics

MATH-026 Applied Calculus Final Examination
December 12, 2006

Show all work. *The time for this examination is 2 hours*

This examination consists of two parts

Part I: Do all questions

1. [20 points]

Find the limits:

(a) $\lim_{x\to 2} \frac{x^2-4}{2x^2-3x-2}$

(b) $\lim_{x\to +\infty} \frac{x^3+2x+7}{2x^3-2x+7}$

(c) $\lim_{x\to 1}(x^2-3x+2)$

2. [20 points]

Use the techniques of differentiation to find the derivative of each of the following:

(a) $f(x) = \frac{x^2-2x}{2x-1}$

(b) $f(x) = 2x(2x^2-3)^3$

(c) $y = e^{2x} + 2x$

(d) $y = \ln(x^2+4)$

3. [20 points]

(a) Use implicit differentiation to find $\frac{dy}{dx}$ if $y^2 + 3xy^2 - 4x^2 = 9$.

Evaluate $\frac{dy}{dx}$ at the point $(-1, 1)$.

(b) Find $\frac{dy}{dx}$ if $y = u^3 - 3u^2 + 1$ and $u = x^2 + 2$.

4. [20 points]

Suppose the total cost in dollars of manufacturing q units of a certain commodity is $C(q) = 3q^2 + q + 500$.

(a) Use marginal analysis to estimate the cost of manufacturing the $41st$ unit.
(b) Compute the actual cost of manufacturing the $41st$ unit.

5. [20 points]

Determine where the function $f(x) = x^4 + 8x^3 + 18x^2 - 8$ is increasing, decreasing, concave upward, and concave downward. Find the relative extrema and inflection points (if any) and sketch the graph.

6. [20 points]

For several weeks, the highway department has been recording the speed of freeway traffic flowing past a certain downtown exit. The data suggest that between 1 : 00P.M. and 6 : 00P.M. on a normal weekday, the speed of the traffic at the exit is approximately $S(t) = t^3 - \frac{21}{2}t^2 + 30t + 20$ miles per hour, where t is the number of hours past noon.
At what time between 1 : 00P.M. and 6 : 00P.M. is the traffic moving the fastest, and at what time is it moving the slowest?

7. [20 points]

A manufacturer finds that in producing x units per day (for $0 < x < 100$), three different kinds of cost are involved:

1. A fixed cost of $1,200 per day in wages,

2. A production cost of $1.20 per day for each unit produced, and

3. An ordering cost of $\frac{100}{x^2}$ dollars per day.

(a) Express the total cost as a function of x.

(b) Determine the level of production that results in a minimal total cost.

8. [20 points]

Find the indicated integrals:

(a) $\int (3x^2 + 2)dx$ (b) $\int 8x(4x^2 - 5)^4 dx$

(c) $\int xe^{2x} dx$

Part II: Do any 4 questions

9. [10 points]

When an electronics store prices a certain brand of stereo at p hundred dollars per set, it is found that q sets will be sold to customers each month, where $q = 40 - 2p^2$.

(a) Find the elasticity of demand for the stereos.

(b) For a unit price of $p = 4$, is the demand elastic, inelastic, or of unit elasticity?

10. [10 points]

Evaluate each definite integral:

(a) $\int_1^4 (3x - 2)dx$ (b) $\int_0^2 \frac{x^2}{(x^3+1)^2} dx.$

11. [10 points]

(a) Find the general solution of the differential equation: $\frac{dy}{dx} = 3x^2 + 5x - 6.$

(b) Find the particular solution of the differential equation satisfying the indicated condition: $\frac{dy}{dx} = \frac{x}{y^2}$; $y = 3$ when $x = 2$.

12. [10 points]

Compute all first order partial derivatives and all second order partial derivatives for the function $f(x, y) = 2xy^5 + 3x^2y + x^2 + 5$.

13. [10 points]

A closed box with a square base is to have a volume of 500 cubic meters. The material for the top and bottom of the box costs \$4 per square meter, and the material for the sides costs \$1 per square meter.

(a) Obtain a formula for the total cost of constructing the box.

(b) Can the box be constructed for less than \$600? Justify your answer.

14. [10 points]

An efficiency study of the morning shift at a certain factory indicates that an average worker arriving on the job at 8 : 00 A.M. will have produced $Q(t) = -t^3 + 8t^2 + 15t$ units t hours later.

(a) Compute the worker's rate of production at 9 : 00 A.M.

(b) At what rate is the worker's rate of production changing with respect to time at 9 : 00 A.M?