Math 026 Applied Calculus
Final Examination
December 9, 2014

Do problems 1 - 12, which are worth up to 15 points each, and one of 13 - 16 for the last 20 points. Show all your work.

1) Find the indicated limit or explain why it does not exist:

a) $\lim_{x\to 1}(x^2 - 5x + 2)$

b) $\lim_{x\to 9}\left(\frac{\sqrt{x}-3}{x-9}\right)$

2) If $f(x) = 2x + 1$ and $g(x) = x^2$, find:

a) $(f \circ g)(x)$;

b) $(g \circ f)(x)$;

c) all values of x for which $(f \circ g)(x) = (g \circ f)(x)$.

3) Give equations for each of the horizontal and vertical asymptotes of

$$f(x) = \frac{x-2}{x^3 - 4x}$$

4) An investment is worth $A(t) = 5000e^{0.06t}$ after t years.

a) How much is the investment worth in 10 years?

b) How many years will it take for the investment to double in value?

5) Find the derivatives:

a) $f(x) = xe^{-3x}$;

b) $g(x) = \frac{x^2+2}{x^2-1}$;

6) Find the derivatives:

a) $h(x) = \ln(x^2 + x + 4)$;

b) $k(t) = \frac{1}{t} + \frac{2}{\sqrt{t}} + 3$.

7) For $f(x) = x^3 - 12x + 3$, determine:

a) the intervals where f is increasing and where it is decreasing;

b) whether each critical point is a maximum, a minimum or neither.

8) For the curve with equation $x^3 - xy + 4y = 1$:

a) Use implicit differentiation to find $\frac{dy}{dx}$;

b) Write the equation of the tangent line to the curve at the point (1,0).

9) Evaluate the integrals:

a) $\int (4x^7 - e^{7x})dx$;

b) $\int \sqrt{4-2x}dx$.

10) Evaluate the integrals:

a) $\int xe^{4x}dx$

b) $\int_1^e t^{-1}dt$

11) Solve the initial value problem (i.e. find the function $y = y(x)$):

$$\frac{dy}{dx} = 3x - 2,\ y = 2 \text{ for } x = -1.$$

12) Find the area bounded by the curve $y = x^2 - x - 2$ and the x axis.

Do one of the following problems for up to 20 points.

13) At the price of p per unit, $q = 10000e^{-0.025p}$ units will be sold.

a) Find the elasticity of demand $-\frac{p}{q}\frac{dq}{dp}$.

b) Find the value of p which maximizes the revenue $R(p) = pq$.

14) If q units of a commodity are produced, they will cost a total of $C(q) = q^2 + 3q + 40$ dollars and can be sold for $p(q) = 11 - q$ dollars each.

a) How many units should be produced to maximize the profit?

b) How many units should be produced to minimize the average cost $A(q) = C(q)/q$?

15) A company produces x units of a product A and y units of a product B each month. The monthly profit, in thousands of dollars, is

$$P(x,y) = -4x^2 + 4xy - 3y^2 + 4x + 10y + 81.$$

a) Find $P_x(1,3)$ and interpret the results.

b) How many of each of A and B should be produced each month to maximize profit? What is the maximum profit?

16) It costs a total of $C(q) = \frac{q^3}{3} - 15q^2 + 200q + 50$ dollars to produce q units of a commodity.

a) What is the marginal cost of the 15th unit?

b) What is the actual cost of the 15th unit?