Math 026 Applied Calculus Final Examination 5/3/2006
Mathematics Department, Howard University

Instructions:

1. Please show all your work, and provide step by step solutions.
2. Each question carries 20 points. Answer any ten.
3. ONLY 10 will be counted for grade.

1. The cost of producing x units at a factory is given by $C(x) = \frac{15x^2+12000}{x+0.03}$. Find the limit of the average cost $C(x)/x$ as $x \to \infty$. i.e, find $\lim_{x\to\infty} \frac{C(x)}{x}$.

2. Find $\frac{dy}{dx}$ or $f'(x)$ in each case. Use implicit differentiation where applicable.
(a) $f(x) = x^3 + \frac{1}{2}x^{2/3} - 2\sqrt{x} + \frac{3}{x} + 7$
(b) $x^2y - 2x^3 + 6 = 2x + 2y$.

3. Let $f(x) = \frac{1}{3}x^3 - 9x + 2$.
(a) Find $f'(x)$ and determine where f is increasing and where f is decreasing.
(b) Use the information in (a) to sketch the graph of f.

4. Find the absolute maximum and minimum of $f(x) = x^5 - 5x^4 + 1$ in the interval $0 \le x \le 5$.

5. The position of a particle $s(t)$ moving on a straight line at time t seconds is given by $s(t) = t^3 - 9t^2 + 15t + 25$.
a) Find the velocity and acceleration of the particle at time t.
(b) Find all times in the first 6 seconds when it is stationary.

6. A manufacturer's total monthly revenue is $R(q) = 240q - 0.05q^2$ when q units are produced and sold during the month. Currently, the manufacturer is producing 80 units per month and planning to increase monthly production by 1 unit. Use marginal analysis to estimate the additional revenue that will be generated by the production and sale of the 81st unit. Compare this estimate with the actual additional revenue.

7. Given $f(x) = xe^{-x}$
(a) Find $f'(x), f"(x)$.
(b) Write an equation for the tangent line to the graph of $f(x)$ at $x = 1$.

8. Find the domain where $f(x) = ln(4x - x^2)$ is well defined. In this domain find the relative maxima and minima of $f(x)$.

9. A carmaker makes cars at a cost of 4000 each. If they are priced at $\$p$ each, then he finds that $100000e^{-.001p}$ are sold per year. (a) For what p will he sell 50000 cars per year? (b) Write down the annual profit function $P(p)$ and find the value of p for which it is maximized. (c) How many cars are sold at this price?

10. Find the function whose first derivative is xe^{x^2} and whose graph passes through $(0, 1)$.

11. Compute $\int_0^1 \frac{x+2}{x^2+4x+2} dx$.

12. Integrate by parts: $\int_2^3 x\ ln(x) dx$. Find the area under the curve $y = x\ ln(x)$ from $x = 2$ to $x = 3$.

13. Solve the differential equation $dy/dx = xy$ with the intial condition $y(0) = 1$.

14. Approximate $\int_0^1 (x^2+x) dx$ using trapezoidal method with four intervals. Compare your answer with actual value of integral obtained using fundamental theorem of calculus.

15. Compute all second order partial derivatives (including mixed partials) of x^4y^3.