Calculus I (Math 156) Fall 2014 Final Exam
Howard University Department of Mathematics
9 December 2014

Name: _____

Show all your work in the bluebook provided. No work, No credit. Best Wishes!

PART I: Do all of the problems in part I.

| 20points | 1. Find the limit or show that it does not exist. Use L'Hospital's Rule where applicable.

(a) $\lim\limits_{x \to 0} \left(\dfrac{\cos x - \cos 3x}{x^2} \right)$

(b) $\lim\limits_{x \to 0} (1 - 2x)^{\frac{1}{x}}$

(c) $\lim\limits_{x \to 5} \dfrac{x^2 - 5x + 6}{x - 5}$

(d) $\lim\limits_{x \to 1} \left(\dfrac{1}{x - 1} + \dfrac{1}{x^2 - 3x + 2} \right)$

| 20points | 2. Find $\frac{dy}{dx}$ and simplify.

(a) $y = x \arctan\left(\dfrac{x}{2}\right) - \ln(x^2 + 4)$

(b) $y = e^x \sin(x^2 - 1)$

(c) $y = \dfrac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}}$

(d) $y = \cos(a^3 + x^3)$

| 20points | 3. The top of a ladder slides down a vertical wall at a rate of 0.15 m/s. At the moment when the bottom of the ladder is 3 m from the wall, it slides away from the wall at a rate of 0.2 m/s. How long is the ladder?

| 20points | 4. Let the function y be defined implicitly as a function of x by the equation
$yx^4 + xy^4 = 18$.

(a) Verify that the point $P(2, 1)$ is on the graph of y.

(b) Find the derivative $y'(x)$.

(c) Evaluate $y'(2)$.

(d) Find the equation of the line tangent to the graph of y at the point $P(2, 1)$.

20points 5. For the function f defined by $f(x) = 4x^3 + 3x^2 - 6x + 1$, find the following:

   (a) The critical numbers of f.

   (b) The intervals on which f is increasing or decreasing.

   (c) Any local maximum and minimum values of f.

   (d) The intervals of concavity and the inflection points.

20points 6. Evaluate the following integrals.

   $(a) \displaystyle\int \sqrt{x}\sin(1 + x^{\frac{3}{2}})\,dx$ $(b) \displaystyle\int_0^{\frac{\pi}{4}} \sec^2 x\,dx$ $(c) \displaystyle\int \frac{e^{3x} + e^x + 1}{e^{2x}}\,dx$ $(d) \displaystyle\int_{-1}^2 (x^3 - 2x)\,dx$

PART II: Do ONLY FOUR of the following problems below.

20points 7. Let $a(t)$ be the constant function $a(t) = 32$.

   (a) Find an anti-derivative $v(t)$ of $a(t)$.

   (b) Find an anti-derivative $s(t)$ of $v(t)$

20points 8. Find $F'(x)$ and $H'(x)$ if:

   $(a)\ F(x) = \displaystyle\int_{x^2}^1 \frac{\sqrt{1 + t^4}}{t}\,dt$ $(b)\ H(x) = \displaystyle\int_1^x ((t + 1)^{100} - 2t + 1)\,dt$

20points 9. The half-life of cesium-137 is 30 years. Suppose we have a 100-mg sample.

   (a) Find the mass that remains after t years.

   (b) How much of the sample remains after 100 years?

   (c) After how long will only 1 mg remain?

20points 10. Verify that the function satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

   $$f(x) = 2x^2 - 3x + 1, \quad [0, 2]$$

20points 11. Show that f is continuous on $(-\infty, \infty)$.

   $$f(x) = \begin{cases} \sin(x) & \text{if } x < \frac{\pi}{4} \\ \cos(x) & \text{if } x \geq \frac{\pi}{4} \end{cases}$$

20points 12. Use logarithmic differentiation to find $f'(x)$ where;

   $$f(x) = (x + 1)^3 (6 - x)^2 \sqrt[3]{2x + 1}.$$