DEPARTMENT OF MATHEMATICS
FALL 2006
MATH 157 CALCULUS II FINAL EXAMINATION
DECEMBER 12, 2006
DO ALL PROBLEMS

1.[15pts]
Find a positive number k such that the average value of $g(x) = \sqrt{3x}$ on the interval $[0, k]$ is 6.

2.[15pts]
The base of a solid is the region bounded by the function $y = 1 - x$, the x-axis and the y-axis. Cross sections perpendicular to the x-axis are isosceles right triangles where the hypotenuse is the distance from the x-axis to the graph of the function. Find the volume of the solid.

3.[15pts]
The region bounded by the given curves is rotated about the specified axis. Find the volume of the resulting solid:
$y = x^3$ and $y = x^2$ is revolved about the x-axis

4.[15pts]
Find a positive value of a that satisfies: $\int_0^\infty \frac{1}{x^2 + a^2} = 1$

5.[15pts]
Make a suitable u-substitution and use integration by parts to evaluate the integral: $\int_0^1 e^{-\sqrt{x}}\,dx$

6.[15pts]
Evaluate the integral $\int_0^{\pi/2} \sin^2 \frac{x}{2} \cos^2 \frac{x}{2}\,dx$

7.[20pts]
(a) Evaluate the integral: $\int \frac{2x^2 + 3x + 3}{(x+1)^3}\,dx$

(b) Evaluate the integral: $\int \frac{x^2}{\sqrt{4-x^2}}dx$

8.[20pts]

Find the radius of convergence and the interval of convergence of the series: $\sum_{k=0}^{\infty}(-1)^k \frac{(x-2)^k}{k}$

9.[20pts]

(a) Determine whether the series converges, and if so, find its sum $\sum_{k=1}^{\infty} 2^{2+k} 5^{1-k}$

(b) Use any method to determine whether the series converges: $\sum_{k-1}^{\infty} \frac{\ln k}{e^k}$

10.[10pts]

Starting with the Maclaurin series $\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k, \ -1 < x < 1$ find the Maclaurin series for $\ln(1+x), -1 < x < 1$.

11.[20pts]

(a) Verify that the series $\sum_{k=1}^{\infty}(-1)^{k+1}\frac{1}{k}$ satisfies the hypotheses of the alternating series test.

(b) Let S be the sum of the series in part (a), find a value of n for which the nth partial sum is ensured to approximate S to the stated accuracy: $|error| < 0.0001$

12.[20pts]

Find the area of the region inside the circle $r = 3\sin\theta$ and outside the cardioid $r = 1 + \sin\theta$