Calculus I Final Exam - inperson
Howard University Mathematics Department

April 30, 2024

MUST GIVE STEP BY STEP EXPLANATIONS TO GET CREDIT FOR ANSWERS.
No calculators or electronic devices are permitted.

PART I: ***Do all three problems. EACH WORTH 24 POINTS.***

1. Given the function $f(x) = 3x^4 + 4x^3$.
   (a) Find the critical points.
   (b) Find the open interval(s) on which the function is increasing or decreasing.
   (c) Apply the First Derivative Test to identify all local extrema.
   (d) Determine the open intervals on which the graph of $f(x)$ is concave upward or concave downward and the point(s) of inflection.
   (e) Locate and identify the absolute extreme values of $f(x)$ on the interval $[-2, 1]$ using Closed Interval Method.
   (f) Use the information in parts (a) through (d) to sketch the graph of $y = f(x)$.

2. Use *limit definition* to find the derivative of -

   a) $f(x) = 4 + 8x - 5x^2$ b) $f(x) = \dfrac{2}{x^2}$

3. The region R is bounded by the x-axis, the curve $y = 2x + 2$, the line $x = 0$, and the line $x = 1$.
   (a) Graph the function over the given interval.
   (b) Sketch the rectangles associated with the Riemann sum $\sum_{i=1}^{4} f(x_i)\Delta x$, where x_i is the right end points. Identify if the approximation is over estimation or under estimation.
   (c) Find a formula for the Right Riemann sum obtained by dividing the interval into n equal subintervals $\left(\text{Hint:} \sum_{i=1}^{n} i = \dfrac{n(n+1)}{2}\right)$.
   (d) Take a limit of the sums obtained in part (c) as $n \to \infty$ to calculate the area under the curve over $[0, 1]$.

PART II: ***Choose any 8 problems. EACH WORTH 16 POINTS.***

1. Let f be a function defined as follows:

$$f(x) = \begin{cases} x^2 + 3x, & \text{if } x < 1 \\ 4, & \text{if } x = 1 \\ 5x - 2, & \text{if } x > 1 \end{cases}$$

   (a) Find $\lim_{x \to 1^-} f(x)$.   (b) Find $\lim_{x \to 1^+} f(x)$.
   (c) Find $\lim_{x \to 1} f(x)$ if it exists. If it does not exist, explain the reason.
   (d) Is f continuous at $x = 1$? Explain the reason to your answer.

2. Find the slope of the tangent line of the function $f(x) = x^4 - 6x + 3$ at $(2, 7)$. Also, find the equation of the tangent line.

3. Find the horizontal and vertical asymptotes of each of the following curves:
   (a) $f(x) = \dfrac{2x^2 + 4x - 1}{x^2 + 4x - 12}$.
   (b) $f(x) = \dfrac{2x^2 - 3x - 9}{x^2 - 5x - 14}$.

   Justify your work for each by computing a limit.

4. (a) Find a suitable function $f(x)$ to approximate $e^{0.015}$.
   (b) Find the linearization to the function obtained in part (a) at $a = 0$.
   (c) Approximate $e^{0.015}$ using linear approximation.

5. A ladder 25 feet long is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 1 feet per second. Consider the triangle formed by the side of the house, the ladder, and the ground. Find the rate at which the area of the triangle is changing when the base of the ladder is 6 feet from the wall.

6. Find the two x-intercepts of $f(x) = x^2 - 3x + 2$ and show that $f'(x) = 0$ at some point between the x-intercepts (Rolle's theorem).

7. Determine whether it is TRUE or FALSE for each of the following statements. No explanation needed.
   (a) If f is differentiable at a, then f is continuous at a.
   (b) A function can have two different slant asymptotes.
   (c) The equation $x^3 - 2x - 1 = 0$ has a solution in the interval $[0, 3]$.

8. Evaluate the following limits using an appropriate method:

   a) $\lim_{x\to 3^+} \frac{7}{x-3}$

   b) $\lim_{x\to 3^-} \frac{7}{x-3}$

   c) $\lim_{x\to 1} \frac{4x^2+3x-2}{3x+2}$

   d) $\lim_{x\to 0^+} \left(\frac{2x+1}{x} - \frac{1}{\sin x}\right)$

9. Use appropriate derivative rule/method to find the first derivative of each of the following functions and justify your solution.

   a) $y^3 + y^2 - 5y - x^2 = -4$

   b) $y = 2x^x, x > 0$

   c) $f(x) = \sqrt{1-x^2}$. Find the point on the curve $f(x)$ where the tangent line is horizontal.

10. A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce a box with maximum volume? Use the optimization method and justify your solution.

11. (a) Evaluate the definite integral: $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1-u^2}{u} du$.

    (b) Let $g(x) = \int_1^{x^3} \frac{\ln t}{t} dt$. Find $g'(x)$.

12. Evaluate the following integrals:

    (a) $\int \left(3x^2 - 2\sin(x) + \frac{1}{x+1} + e^{2x}\right) dx$.

    (b) $\int_0^{\pi/2} \sin^2(x)\cos(x) dx$.