Calculus I Final Exam - A
Howard University, Department of Mathematics
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MUST GIVE STEP BY STEP EXPLANATIONS TO GET CREDIT FOR ANSWERS
No calculators or electronic devices are permitted

PART I: **Do all three problems**. **EACH WORTH 24 POINTS**

1. (a) Let $f(x) = \begin{cases} \sqrt{-4x} & \text{if} \quad x < 0 \\ 4 - x & \text{if} \quad 0 \leq x < 4 \\ (x-4)^2 & \text{if} \quad x > 4 \end{cases}$

Evaluate each limit, if it exists

(i) $\lim_{x \to 0^+} f(x)$, (ii) $\lim_{x \to 0^-} f(x)$, (iii) $\lim_{x \to 0} f(x)$
(iv) $\lim_{x \to 4^+} f(x)$, (v) $\lim_{x \to 4^-} f(x)$, (vi) $\lim_{x \to 4} f(x)$

(b) If $f(x) = \sqrt{1-3x}$, use the definition of a derivative to find $f'(x)$
(c) Find the domains of f and f'

2. For the function $f(x) = x^3 - 3x^2 - 9x + 4$
(a) Find the critical numbers
(b) Find the intervals of increase or decrease
(c) Find the local maximum and minimum values
(d) Find the intervals of concavity and the inflection points

3. (a) If $f(x) = x^2 - 2x, \quad 0 \leq x \leq 3,$ evaluate the Riemann sum with $n = 6$, taking the sample points to be the right endpoints. What does the Riemann sum represent?Illustrate with a diagram.
(b) Evaluate the integral $\int \left(x^e + \sec^2 x + \sec x \tan x\right)\ dx$
(c) Find the derivative of $G(x) = \int_0^{x^3} e^{-t^2}\ dt$

PART II: Choose any 8 problems. EACH WORTH 16 POINTS

4. (a) Find the domain of the function $f(x) = \sqrt{x-3}$
(b) If $f(x) = x^2 + 2x + 3$, find the difference quotient
$\dfrac{f(x+k) - f(x)}{k},\ k \neq 0$

5. You are building a rectangular enclosure out of fencing that you will pay for by the meter. The enclosure must have an area of 500 square meters. Determine the dimensions of the enclosure that will minimize the cost of materials. Hint: If your are minimizing cost, you are minimizing the meters of fence you must purchase.

6. Find the following limits

(i) $\lim_{x\to 0} \dfrac{\sin 4x}{\tan x}$ (ii) $\lim_{x\to\infty} \dfrac{\ln x}{\sqrt[3]{x}}$ (iii) $\lim_{x\to 1} \dfrac{x^3-2x^2+1}{x^3-1}$

7. Evaluate the definite integrals
(i) $\int_0^1 x^2(x^3+1)^4\ dx$ (ii) $\int_1^2 x\sqrt{x-1}dx$

8. (a) Find f if $f''(x) = 2+\cos x, \quad f(0) = -1, \quad f(\pi/2) = 0$
(b) Find the absolute maximum and absolute minimum for the function $f(x) = 2x^3 - 3x^2 - 12x + 1 \quad$ on $\quad [-2, 3]$

9. The function $f(x) = x^2 - x + 1$ is continuous and differentiable for all real numbers. Therefore, $f(x)$ satisfies the hypotheses of the Mean Value Theorem on $[0, 2]$. Determine the point c in $[0, 2]$ at which the instantaneous rate of change of $f(x)$ is equal to the average rate of change of $f(x)$ over $[0, 2]$

10. A particle moves with position function:
$s(t) = t^4 + 4t^3 - 20t^2 + 10t, \qquad t \geq 0$
(a) At what time does the particle have a velocity of $10m/s$
(b) At what time is the acceleration 0? What is the significance of this value of t?

11. (a) Find $\dfrac{dy}{dx}$ by implicit differentiation: $x^2 - xy + y^2 = 1$
(b) Find an equation of the tangent line to the curve $x^2 - xy + y^2 = 1$ at $(0, 1)$

12. Evaluate the indefinite integrals
(i) $\int \dfrac{\sin x}{1+\cos^2 x}dx$ (ii) $\int \dfrac{dx}{x\sqrt{\ln x}}$ (iii) $\int \dfrac{x^2 dx}{x^2+1}$