**Math 158** **Calculus III** **Final Exam Form A**
**12/5/2025**

**SHOW ALL WORK. Justify your answers! No Calculator Permitted.**
**Simplify your answers. Give exact answers whenever possible.**
**Each problem = 20 pts. Solve all 10 problems below. Exam total=200 pts.**

1. A particle traveling in a circle has velocity function $v(t) = 3\sin t\mathbf{i} + 3\cos t\mathbf{j}$ with initial position $\mathbf{r}(0) = 3\mathbf{j} - 3\mathbf{k}$.
   (a) Find the position function $\mathbf{r}(t)$ for this particle.
   (b) Find the distance traveled by this particle over $0 \le t \le \frac{\pi}{2}$.

2. Do the following, where $z = f(x, y)$ where $x = r\cos 2\theta$, $y = r\sin 2\theta$.
   (a) Using limit notation, write what it means for the function $f_y$ to be continuous at the point $(a, b)$.
   (b) Find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$ if $f_x$ and $f_y$ are continuous.
   (c) Show that $y = r\sin 2\theta$ is a solution to the partial differential equation $4(y_r)^2 + (y_{\theta r})^2 + 4(y_{r\theta})^2 + (y_{r\theta\theta})^2 = 20$.

3. Let $f(x, y) = xy + e^{x+3y}$ denote the temperature of a circular disk in °F. Assume distance is measured in feet.
   (a) Find the equation of the tangent plane to $z = f(x, y)$ at $(-3, 1)$.
   (b) Find $D_u f(-3, 1)$ in the direction of $\langle 1, 4\rangle$, indicate units of measure, and explain what $D_u f(-3, 1)$ means.

4. The plane $z = 2x + 8y$ intersects the cylinder $x^2 + y^2 = 17$ in an ellipse. Find the maximum and minimum perpendicular distances from this ellipse to the xy-plane.

5. Let $D$ be the triangular region in the xy-plane with vertices at $(0, 0, 0)$, $(2, 2, 0)$, and $(4, 0, 0)$, which is bounded by $y = x$, $x + y = 4$ and $y = 0$. Do the following.
   (a) Show that $\iint_D y \, dA = \frac{8}{3}$ by evaluating this integral.
   (b) Describe a solid which the integral in (a) represents the volume of.
   (c) If $C$ is the boundary of $D$ oriented counter-clockwise, use Green's Theorem and your answer in part (a) to evaluate the line integral $\int_C x^2 \, dx \, + \, xy \, dy$.
   (d) Let $-C$ be the traversal of $C$ in a clockwise direction. Use your answer in (c) to set up and state the value of a line integral over $-C$ that equals the work done by $\mathbf{F} = x^2\mathbf{i} + xy\mathbf{j}$ in moving a particle along $-C$ (in a clock-wise direction).

6. Find the area of the part of the paraboloid $z = 9 - x^2 - y^2$ that lies above the xy-plane.

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7. Let $E$ be the solid which is under the parabolic cylinder $z = x^2$ and above the region $D = \{(x, y, 0) \mid 0 \leq y \leq 3,\ \frac{1}{3}y \leq x \leq 1\}$. Do the following.

   (a) Simplify $\displaystyle\iiint_E \cos z \, dV$ into an iterated double integral.

   (b) Switch the order of the integration in the double integral obtained in part (a), and simplify it into a single integral with limits. Do not evaluate.

8. Let $E_1$ be the solid bounded by the ellipsoid $\frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{25} = 1$. Do the following.

   (a) Show that the substitution $x = 3u$, $y = 2v$, and $z = 5w$ transforms $E_1$ into a solid $E_2$ which is bounded by the unit sphere centered at the origin.

   (b) Compute the Jacobian $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ of this change in variables.

   (c) Use the Change in Variables Theorem to rewrite the following triple integral over $E_1$ as a triple integral over $E_2$ in terms of $u$, $v$, and $w$.

$$\iiint_{E_1} \sqrt{\frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{25}} \, dV$$

   (d) Rewrite the triple integral over $E_2$ obtained in part (c) as a product of three single integrals with limits by first changing the integral over $E_2$ to spherical coordinates. Do not evaluate.

9. Let $E$ be the right circular cylinder bounded by the surfaces $x^2 + y^2 = 4$, $z = 3$, and $z = 9$. The velocity field $\mathbf{F} = 3x\mathbf{i} + 2y\mathbf{j} + 4z\mathbf{k}$ describes how a liquid flows outward through the permeable surface $S$ of $E$ equipped with an outward unit normal vector function $\mathbf{n}$. Distance is measured in ft and time in min. Do the following.

   (a) Use Gauss's Divergence Theorem to evaluate $\displaystyle\iint_S \mathbf{F} \cdot \mathbf{n} \, dS$.

   (b) Let $S_1$ and $S_2$ respectively be the top and bottom circular disks of $E$. Evaluate $\displaystyle\iint_{S_1} \mathbf{F} \cdot \mathbf{n} \, dS$ and $\displaystyle\iint_{S_2} \mathbf{F} \cdot \mathbf{n} \, dS$.

   (c) Use the results in part (a) and (b) to find the flux of $\mathbf{F}$ through $S_3$, the part of $S$ bounded by $x^2 + y^2 = 4$. Indicate units of measure, and explain what your answer means in terms of liquid flow.

10. Let $S$ be the rectangle which is the part of the plane $y + z = 2$ that has vertices at $(4, 2, 0), (0, 2, 0), (0, 0, 2)$ and $(4, 0, 2)$ with boundary $C$. Let the force field $\mathbf{F} = 2y\mathbf{i} + 8x\mathbf{j} - 5xz\mathbf{k}$. Do the following.

    (a) Show that $\nabla \times \mathbf{F} = 5z\mathbf{j} + 6\mathbf{k}$.

    (b) State the equation for Stokes' Theorem and use it to set up a simplified, iterated double integral that equals the work done by $\mathbf{F}$ in moving a particle counter-clockwise around $C$. Do not evaluate.

**(end of exam)**