Math 158 **Calculus III** **Final Exam Form B**
12/5/2025

SHOW ALL WORK. Justify your answers! No Calculator Permitted.
Simplify your answers. Give exact answers whenever possible.
Each problem = 20 pts. Solve all 10 problems below. Exam total=200 pts.

1. A particle traveling in a circle has velocity function $v(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j}$ with initial position $\mathbf{r}(0) = 2\mathbf{j} + 2\mathbf{k}$.
   (a) Find the position function $\mathbf{r}(t)$ for this particle.
   (b) Find the distance traveled by this particle over $0 \le t \le \frac{\pi}{2}$.

2. Do the following, where $z = f(x, y)$ where $x = r\cos 3\theta$, $y = r\sin 3\theta$.
   (a) Using limit notation, write what it means for the function f_x to be continuous at the point (a, b).
   (b) Find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial \theta}$ if f_x and f_y are continuous.
   (c) Show that $x = r\cos 3\theta$ is a solution to the partial differential equation $9(x_r)^2 + (x_{\theta r})^2 + 9(x_{r\theta})^2 + (x_{r\theta\theta})^2 = 90$.

3. Let $f(x, y) = xy + e^{2x+y}$ denote the temperature of a circular disk in °F. Assume distance is measured in feet.
   (a) Find the equation of the tangent plane to $z = f(x, y)$ at $(-1, 2)$.
   (b) Find $D_u f(-1, 2)$ in the direction of $\langle 3, 5\rangle$, indicate units of measure, and explain what $D_u f(-1, 2)$ means.

4. The plane $z = 4x + 6y$ intersects the cylinder $x^2 + y^2 = 13$ in an ellipse. Find the maximum and minimum perpendicular distances from this ellipse to the xy-plane.

5. Let D be the triangular region in the xy-plane with vertices at $(0, 0, 0)$, $(1, 1, 0)$, and $(2, 0, 0)$, which is bounded by $y = x$, $x + y = 2$ and $y = 0$. Do the following.
   (a) Show that $\displaystyle\iint_D y\, dA = \frac{1}{3}$ by evaluating this integral.
   (b) Describe a solid which the integral in (a) represents the volume of.
   (c) If C is the boundary of D oriented counter-clockwise, use Green's Theorem and your answer in part (a) to evaluate the line integral $\displaystyle\int_C x\, dx + xy\, dy$.
   (d) Let $-C$ be the traversal of C in a clockwise direction. Use your answer in (c) to set up and state the value of a line integral over $-C$ that equals the work done by $\mathbf{F} = x\mathbf{i} + xy\mathbf{j}$ in moving a particle along $-C$ (in a clock-wise direction).

6. Find the area of the part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the xy-plane.

(continued on the next page)

7. Let E be the solid which is under the parabolic cylinder $z = x^2$ and above the region $D = \{(x, y, 0) \mid 0 \le y \le 2,\ \frac{1}{2}y \le x \le 1\}$. Do the following.

   (a) Simplify $\iiint\limits_E \cos z \, dV$ into an iterated double integral.

   (b) Switch the order of the integration in the double integral obtained in part (a), and simplify it into a single integral with limits. Do not evaluate.

8. Let E_1 be the solid bounded by the ellipsoid $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} = 1$. Do the following.

   (a) Show that the substitution $x = 2u$, $y = 3v$, and $z = 4w$ transforms E_1 into a solid E_2 which is bounded by the unit sphere centered at the origin.

   (b) Compute the Jacobian $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ of this change in variables.

   (c) Use the Change in Variables Theorem to rewrite the following triple integral over E_1 as a triple integral over E_2 in terms of u, v, and w.

$$\iiint\limits_{E_1} \sqrt{\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16}} \, dV$$

   (d) Rewrite the triple integral over E_2 obtained in part (c) as a product of three single integrals with limits by first changing the integral over E_2 to spherical coordinates. Do not evaluate.

9. Let E be the right circular cylinder bounded by the surfaces $x^2 + y^2 = 9$, $z = 1$, and $z = 5$. The velocity field $\mathbf{F} = 2x\mathbf{i} + 3y\mathbf{j} + 4z\mathbf{k}$ describes how a liquid flows outward through the permeable surface S of E equipped with an outward unit normal vector function $\mathbf{n}$. Distance is measured in ft and time in min. Do the following.

   (a) Use Gauss's Divergence Theorem to evaluate $\iint\limits_S \mathbf{F} \cdot \mathbf{n} \, dS$.

   (b) Let S_1 and S_2 respectively be the top and bottom circular disks of E. Evaluate $\iint\limits_{S_1} \mathbf{F} \cdot \mathbf{n} \, dS$ and $\iint\limits_{S_2} \mathbf{F} \cdot \mathbf{n} \, dS$.

   (c) Use the results in part (a) and (b) to find the flux of $\mathbf{F}$ through S_3, the part of S bounded by $x^2 + y^2 = 9$. Indicate units of measure, and explain what your answer means in terms of liquid flow.

10. Let S be the rectangle which is the part of the plane $y + z = 4$ that has vertices at $(2, 4, 0), (0, 4, 0), (0, 0, 4)$ and $(2, 0, 4)$ with boundary C. Let the force field $\mathbf{F} = 4y\mathbf{i} + 6x\mathbf{j} - 3xz\mathbf{k}$. Do the following.

   (a) Show that $\nabla \times \mathbf{F} = 3z\mathbf{j} + 2\mathbf{k}$.

   (b) State the equation for Stokes' Theorem and use it to set up a simplified, iterated double integral that equals the work done by $\mathbf{F}$ in moving a particle counter-clockwise around C. Do not evaluate.

(end of exam)